Legendre Spectrum for texture classification

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Abstract

This paper deals with texture classification using a multifractal approach. More precisely we analyse the singularity/regularity exponent that compose the textures because they theoretically carry most of the information. The analysis is made using the Legendre spectrum.

Then a parameter vector is computed to describe this spectrum in order to classify the textures with an unsupervised k-means classifier. The resulting algorithm is evaluated against a classification directly based on the textures.

Key words: fractal, classification, texture analysis, statistical analysis

1. Introduction

Texture classification is a complex problem because there isn't any precise definition of what is a texture. In fact, textures are defined and studied in different ways depending on the application. This mainly leads to statistical and structural methods [1].

The choice of classification methods is also not an easy task because of the great intensity variability of textures. A lot of approaches are present in the literature as filtering methods [2, 3, 4], mathematical morphology for structural analysis [5, 6], and fractal analysis [7].

Grammatical approaches are also used to extract texture construction rules. Other methods extract spatial relations between grey levels, such as run lengths method introduced by Galloway [8], co-occurrence matrices [9]. Also authors consider textures as random processes and build autoregressive models [10], Markov models [11] and so on.

In this article, we will focus on natural textures classification, and we will consider that they can be considered as random textures. We will show in section 6 that the use of Legendre spectrum [12] is of great interest for texture classification, compared to classification over classical texture features.

Moreover, we will show that the classification based on Legendre Spectrum is more robust to the size of the texture sample (within some limits depending on the coarseness of the texture) which is a workable result for the texture classification community as well as for people interested in Legendre spectrum.

2. Multi fractal background

The multifractal theory is the result of works from measure theory, dynamical system theory and physic statistics. It is considered as a formalism for the analysis and the characterisation of signals with a geometrical and statistical approach.

The theory is linked to the singularity spectrum estimation of a mathematical measure having wide variations. More precisely, a simple way to describe a signal is to determine its spectrum. The first step is to find the adapted measure and next to compute the singularity exponent.

In the following, we note T a NxN texture region having a compact support E and values in the interval [0,…,G]. $T(i, j) : E \rightarrow [0,\dots,G]$, where $(i,j) \in N^2$ is the texture coordinate. We apply on T the method defined by Stanczyk and Sharpe (1999) [13] and the Legendre transform to estimate the multifractal spectrum (denoted Legendre Spectrum).

3. Legendre Spectrum estimation

Let us consider a Borel measure μ defined on $[0,1]\times[0,1]$ and ν_n an increasing sequence of positive integers. Let us define $I_n(i, j)$ as follow

$$I_n(i, j) = \left[\frac{i}{\nu_n}, \frac{i+1}{\nu_n}\right] \times \left[\frac{j}{\nu_n}, \frac{j+1}{\nu_n}\right]$$

We choose $\nu_n = 2^n$ in order to have a dyadic sequence with $n = 1, 2, \dots, \log_2(N/2)$.

Based on thermodynamic statistics analogy Halsey and al. [12] propose to compute the singularity spectrum from a kind of free energy $\tau(q)$.

For all $q \in \Re$, the following limit exist:

$$\tau(q) = \lim_{n \to \infty} \frac{\log\left(\sum_i \sum_j \mu[I_n(i, j)]^q\right)}{\log(\nu_n)}$$

with

$$\mu[I_n(i, j)]^q = \frac{[P_n(i, j)]^q}{\sum_i \sum_j [P_n(i, j)]^q}$$

Where $P_n(i, j)$ is the probability estimation in a ball of radius ν_n. Let us note that for each q value, μ is defined on $[0, 1]\times[0, 1]$.

Now we estimate the singularity exponents, $\alpha(q)$, and the Legendre Spectrum, $f(q)$, using the method developed by Chhabra and Jensen (1989) [14]. They directly compute these values with a linear regression applied on both following formula

$$\alpha(q) = \lim_{n \to \infty} \frac{\sum_i \sum_j \mu[I_n(i, j)]^q * \log[P_n(i, j)]}{\log(\nu_n)}$$

and

$$f(q) = \lim_{n \to \infty} \frac{\sum_i \sum_j \mu[I_n(i, j)]^q * \log\{\mu[I_n(i, j)]^q\}}{\log(\nu_n)}$$

These estimations are respectively obtained by a derivation and a Legendre Transform on $\tau(q)$.

$$\alpha(q) = \frac{d\tau(q)}{dq} \quad \text{and} \quad f[\alpha(q)] = \inf_{q \in \Re}[q\alpha - \tau(q)]$$

4. Used parameters

In order to classify the textures we have to characterize them. The characterization based on the statistical properties directly computed on the grey level histogram is commonly used and remind on table 1. These parameters are computed using moments [15, 16].

In table 1. Z_i is a random variable indicating intensity and $p(Z_i)$ the corresponding histogram value.

Statistical parameters	
Name	Expression
Mean	$m = \sum_{i=0}^{L-1} Z_i * p(z_i)$
Standard deviation	$\sigma = \sqrt{M_2}$
Smoothing	$L = 1 - 1/(1+\sigma^2)$
3 order moment	$M_3 = \sum_{i=0}^{L-1} (Z_i - m)^3 * p(Z_i)$
Regularity	$R = \sum_{i=0}^{L-1} [p(Z_i)]^2$
Entropy	$e = -\sum_{i=0}^{L-1} p(Z_i) * \log_2[p(Z_i)]$

Table 1. Name and expression of parameters computed on texture

We introduce another way to characterize a texture using multifractal parameters (Table 2) computed on singularity exponent (α) and Legendre spectrum ($f(\alpha)$).

Some theoretical results [17] show that the information given by the singularity exponent is more pertinent than the one given by the texture itself. The Legendre Spectrum is a way to characterize these exponents.

Multifractal parameters	
Name	Expression
Surface	$\sum_{i,\, f(q_i) \& f(q_{i+1}) > a} \frac{\{[f(q_i) + f(q_{i+1})] * [\alpha(q_{i+1}) - \alpha(q_i)]\}}{2}$
α_{min}	$\min(\alpha)$
α_{max}	$\max(\alpha)$
$f(\alpha_{min})$	$f(\min(\alpha))$
$f(\alpha_{max})$	$f(\max(\alpha))$
Width	$\lvert f(\max(\alpha)) - f(\min(\alpha)) \rvert$

Table 2. Name and expression of parameters computed on Legendre Spectrum

The figure 1. shows the Legendre spectrum (red curve) and summaries the features described in Table 2.

The surface parameter has been used in [16] for pattern recognition. As illustrated in figure 1.the studied surface is the upper part of the curve limited by $f(q)=1.8$ [16].

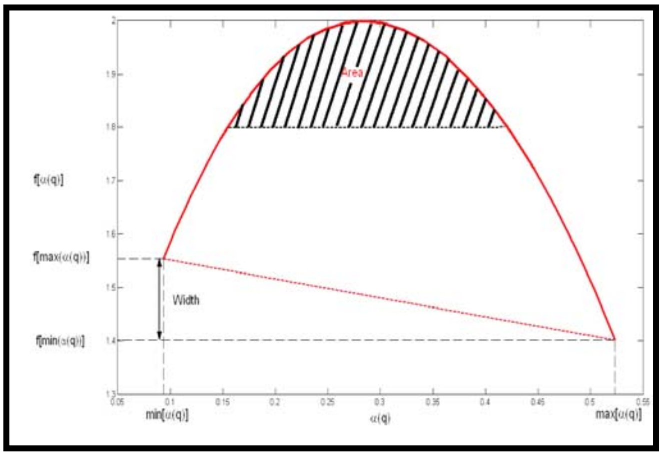

Figure 1. Legendre Spectrum and parameters

5. Data description

The sample textures are built as follow: each reference texture is an image of size 512×512. We extract sixteen 128×128 samples and sixty four 64×64 samples.

The textures used for the illustration are five natural textures from the Brodatz album [18] (Figure 2) and five forest textures extracted from IKONOS images (Figure 3).

The forest textures have been chosen because we must treat them in the framework of the CESAR project. They present great visual similarities rendering characterization more difficult.

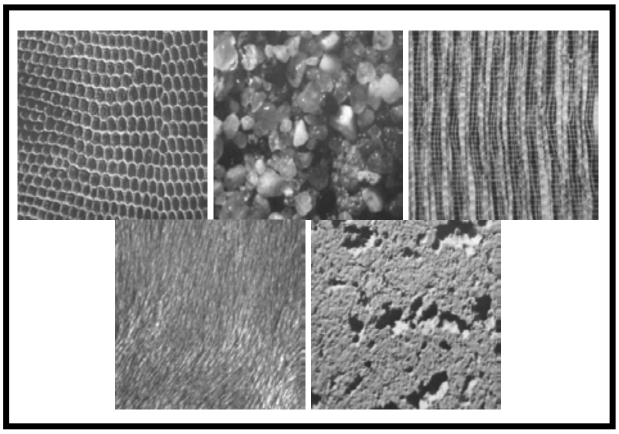

Figure 2. Brodatz album textures

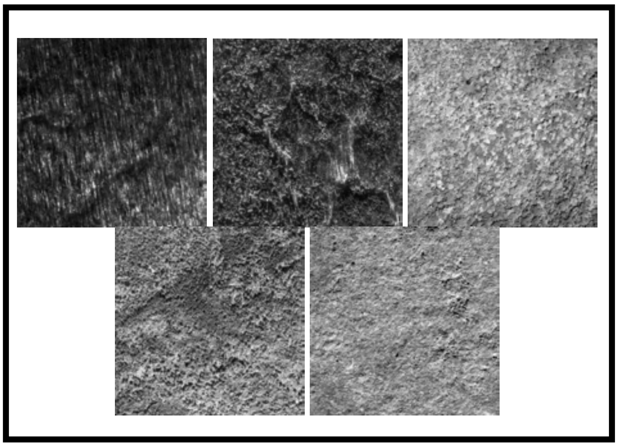

Figure 3. Forest textures extracted from IKONOS images

The first sample set, S_1, is composed of the 512×512 reference textures (1×5) and the 128×128 samples (16×5 samples).
The second sample set, S_2, is composed of the 512×512 reference textures (1×5) and the 64×64 samples (64×5 samples).
The third sample set, S_3, regroups all samples ((1+16+64)×5 samples).

6. Classification results

We compare the results of a k-means classifier applied using statistical parameters presented in Table 1 against those of a k-means classifier applied on our multifractal parameters (Table 2).

In both cases, we use an unsupervised k-means classification where the number of classes is known.

Table 3. shows the results for Brodatz textures and Table 4. the results for IKONOS textures.

Brodatz Textures		S_1	S_2	S_3
k-means classifier applied on	Texture	0.84	0.7294	0.52
	Multifractal spectrum	**0.96**	**0.80**	**0.76**

Table 3. Classification rate obtained with Brodatz textures

IKONOS Textures		S_1	S_2	S_3
k-means classifier applied on	Texture	0.54	0.64	0.48
	Multifractal spectrum	**0.83**	**0.78**	**0.72**

Table 4. Classification rate obtained with Ikonos textures

Both tables show a better classification in favour of multifractal parameters.
One can note that classification on the Brodatz textures is easier than on the IKONOS ones and is linked to the nature of these textures.

Rates obtained with IKONOS textures are more representative of operational results. Indeed, forest textures are quite similar and the choice of characterisation parameters is more critical.

Finally the multifractal spectrum is able to deal with different samples size. Indeed, the classifications applied on S_3 sample sets are acceptable with this spectrum (0.76 and 0.72 rates) while it is not the case with classical features (0.52 and 0.48 rates).

We also note that the classification rate is unstable for IKONOS textures when using the statistical parameters (Table 4.).
This is not the case for multifractal parameters and we observe a decreasing classification rate depending on the size of the texture sample (Table 3. and 4.).

Theoretically, the multifractal spectrum is independent from the sample size. We interpret this result by the fact that these textures are non-silimar.
A close value of the classification rate between S_1 and S_2 could be interpreted by a strong similarity between the texture samples.

7. Conclusion and future works

Texture classification is a complex problem, because there isn't any complete way to characterize a texture. This work aims to use multifractal theory to improve classification. The multifractal parameters that we choose lead to better results than the one directly obtained from the textures.

Classification results can be improved by different ways. Current works try to replace the Legendre spectrum by the large deviation spectrum to compare the results and combine, if possible, the parameters.

Moreover, other parameters could be extracted from these spectrums to complete their description. As examples we could extend the surface or compute the slope of the line joining the two extremity of the curve.

Finally, we will study more precisely the dependence of the multifractal parameters according to the size and the resolution of the sample textures.
Theoretically the multifractal parameters should be independent to the texture sample size but, as underlined in the result section, this is not the case in practice particularly for non-similar textures. Concerning resolution, and always theoretically, multifractal parameters computed over self-similar textures don't depend on the resolution. We will study the impact of multi resolution texture samples on the classification for self-similar textures and for other kind of textures.

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9. References

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