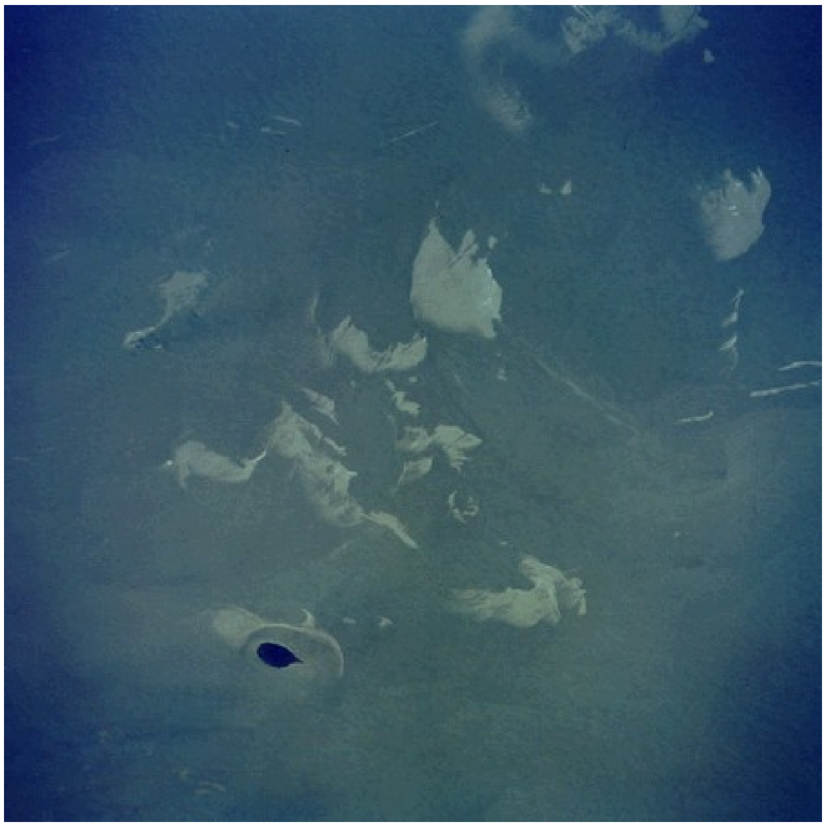
Surfactant Dynamics on Sea Surface

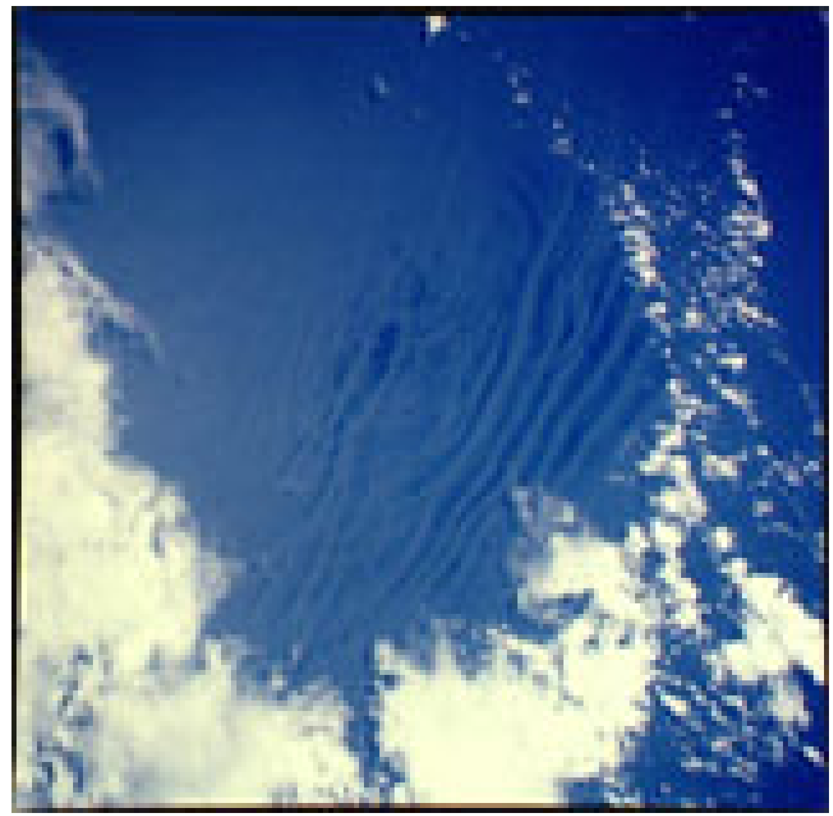
Tatiana Talipova, Institute of Applied Physics, Nizhny Novgorod, Russia

December, 13, 2007

SAR IMAGES

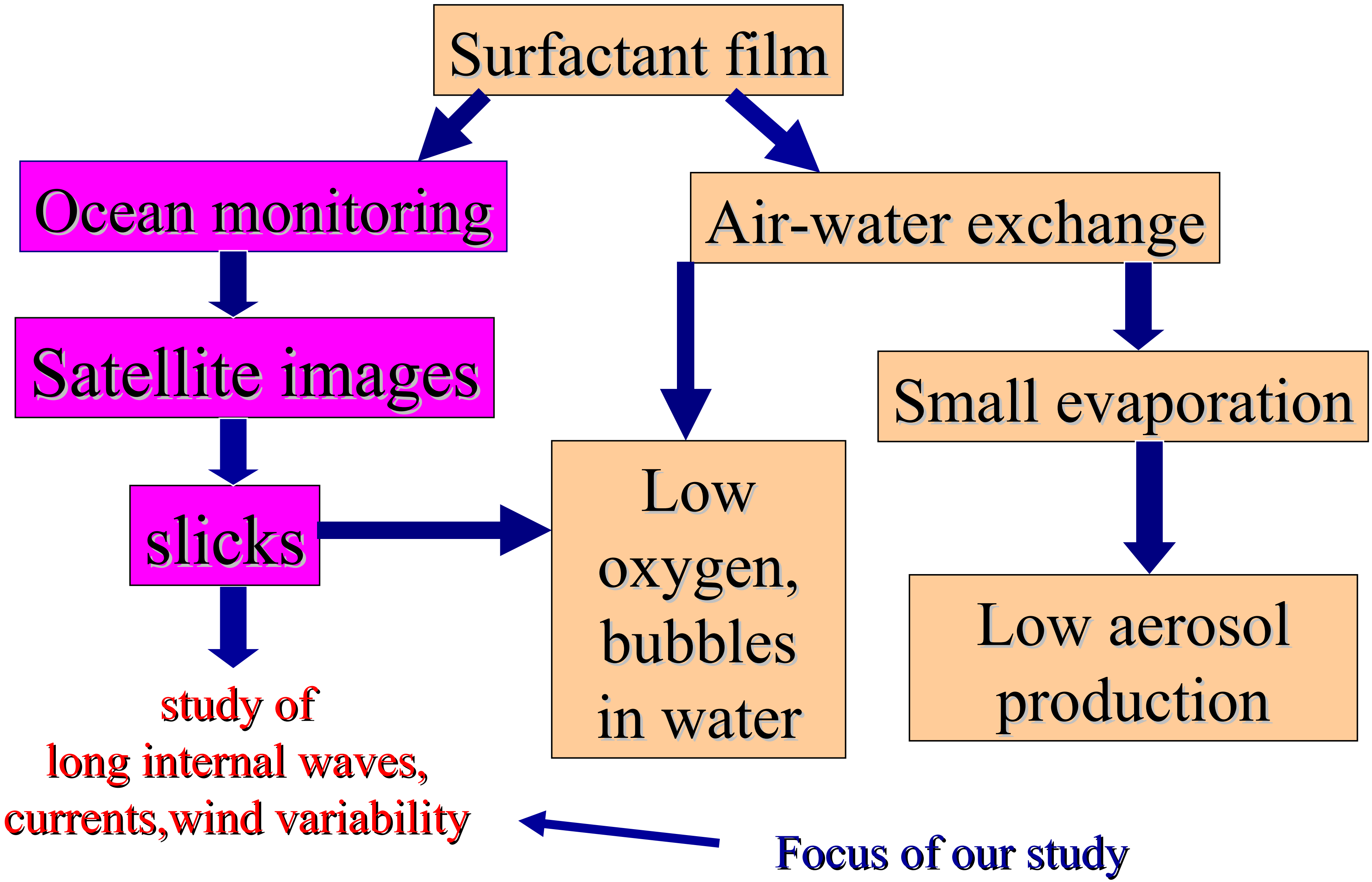


Oil sleek field in Persian Gulf



Surfactants in field of internal waves

MOTIVATIONS:



Surfactant sources
in environment

From Ocean

Biological organic matter:
plankton,
fishes,
drift weed,
organic sediment

From Atmosphere

Aerosol

Transport of Saharan dust over the Caribbean Islands: Study of an event

JGR, V. 110, D18S09, 2005 *R. H. Petit,[1] M. Legrand,[2] I. Jankowiak,[2] J. Molinie´,[1] C. Asselin de Beauville,[1] G. Marion,[1] and J. L. Mansot[3]*

A dust plume transported across the Atlantic Ocean from West Africa to Guadeloupe in June 1994 is studied using several complementary and cross-checking techniques. During this event the dust optical depth measured in Guadeloupe was high from 19 to 22 June, peaking at 1. Meteosat-5 IR imagery is used to locate in SW Sahara the source of emitted dust, consistent with the simulated backward trajectories of the dusty air masses arriving over Guadeloupe. Meteosat-3 visible light spectrometer (VIS) imagery over the north tropical Atlantic shows the dust plume leaving the African coast on 15 June and its subsequent spreading over ocean on the following days. The back trajectories indicate a strong uplift from the African source to an altitude of 5000 m on 14 and 15 June, followed by a subsiding motion of the dust plume from the African coast to Guadeloupe, in agreement with the meteorological soundings performed at east and west sides of the Atlantic Ocean. Such uplifts, observed during summer, are shown to be a condition for the long-range transport of dust through the Atlantic. It is also observed that while dust transport is associated with the dynamics of the Saharan air layer, the latter can be dust free. The transported mass of dust was in the range 2.5–5 Mt for this event. Electronic microscopy applied to the mineral particles collected in rainwater just after the dust event shows the predominance of particles larger than 1 mm in the long-range transport from Africa.

Man-made pollution

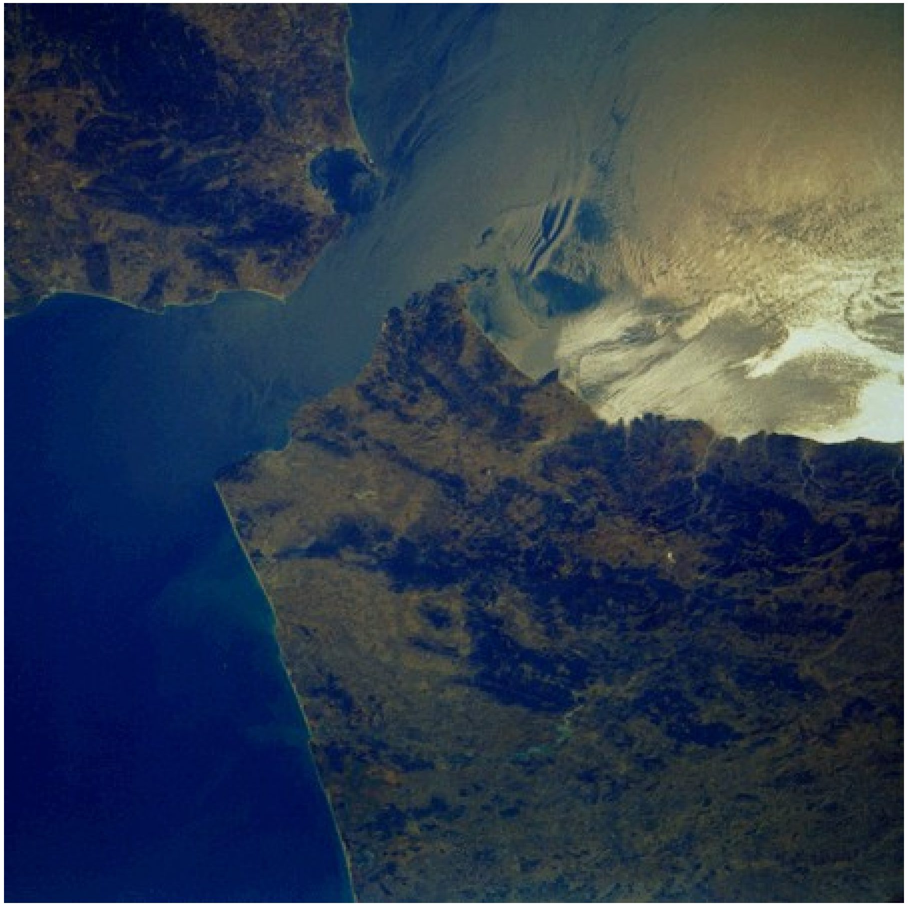
- Ocean beach dumps
- Sewage
- Manufacturing waters
- Oil tanker accidents

Ship Wake

First evidence for the detection of natural surface films by the QuikSCAT scatterometer

I.-I. Lin,[1] Werner Alpers,[2] and W. Timothy Liu[3]

Internal waves in Gibraltar



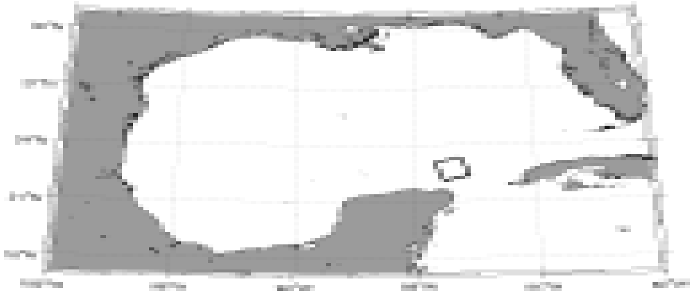
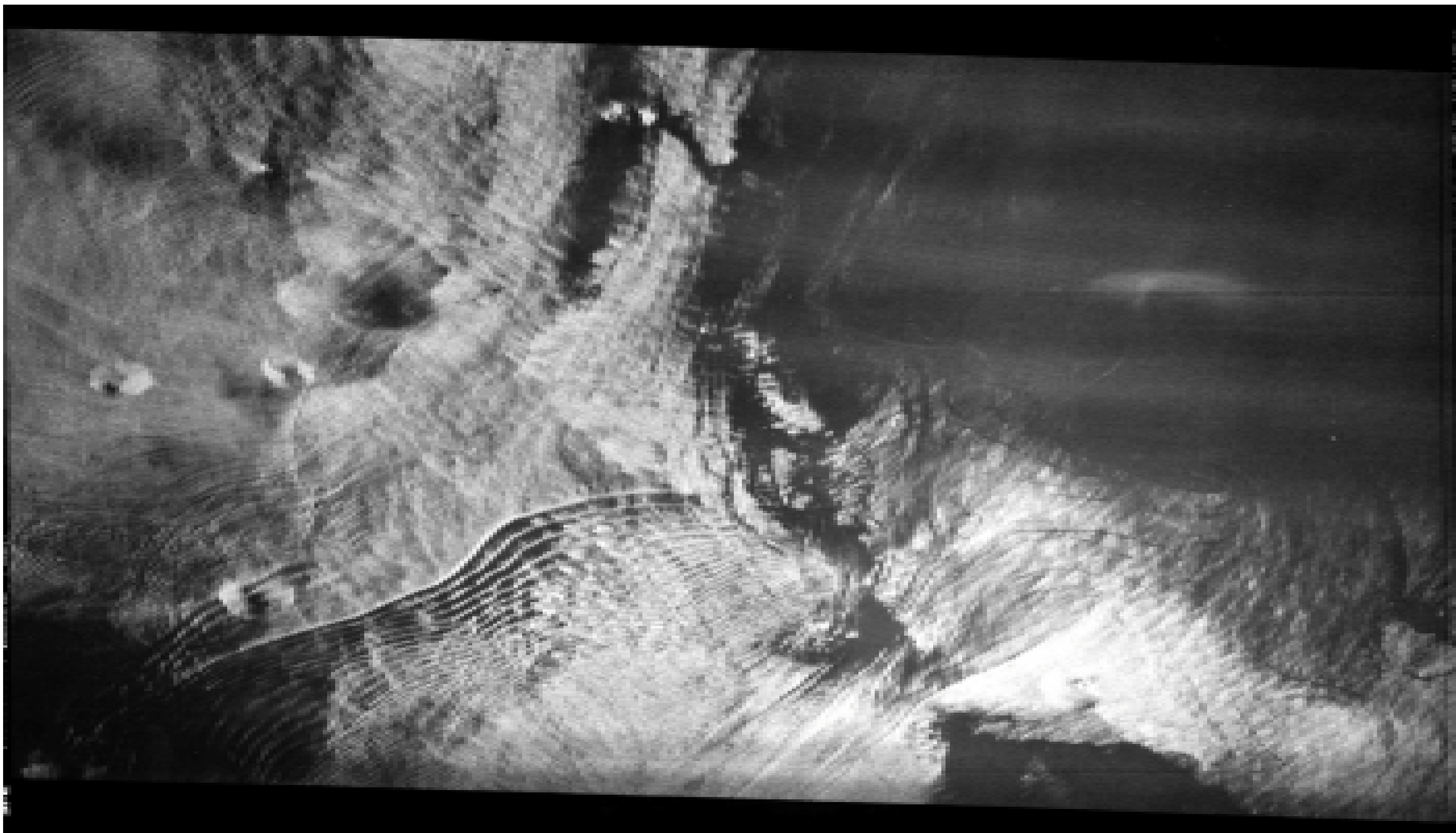


Figure 4. SEASAT (L-band, HH) SAR image of the Yucatan Strait acquired on 24 August 1978 at 0122 UTC (Rev 838). The image shows a very strong signature of an internal wave packet propagating to the northwest over the continental shelf. Imaged area is 100 km x 100 km. [Image courtesy of Ben Holt NASA JPL]



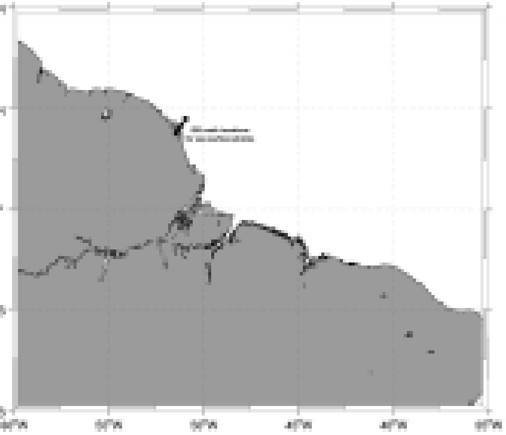
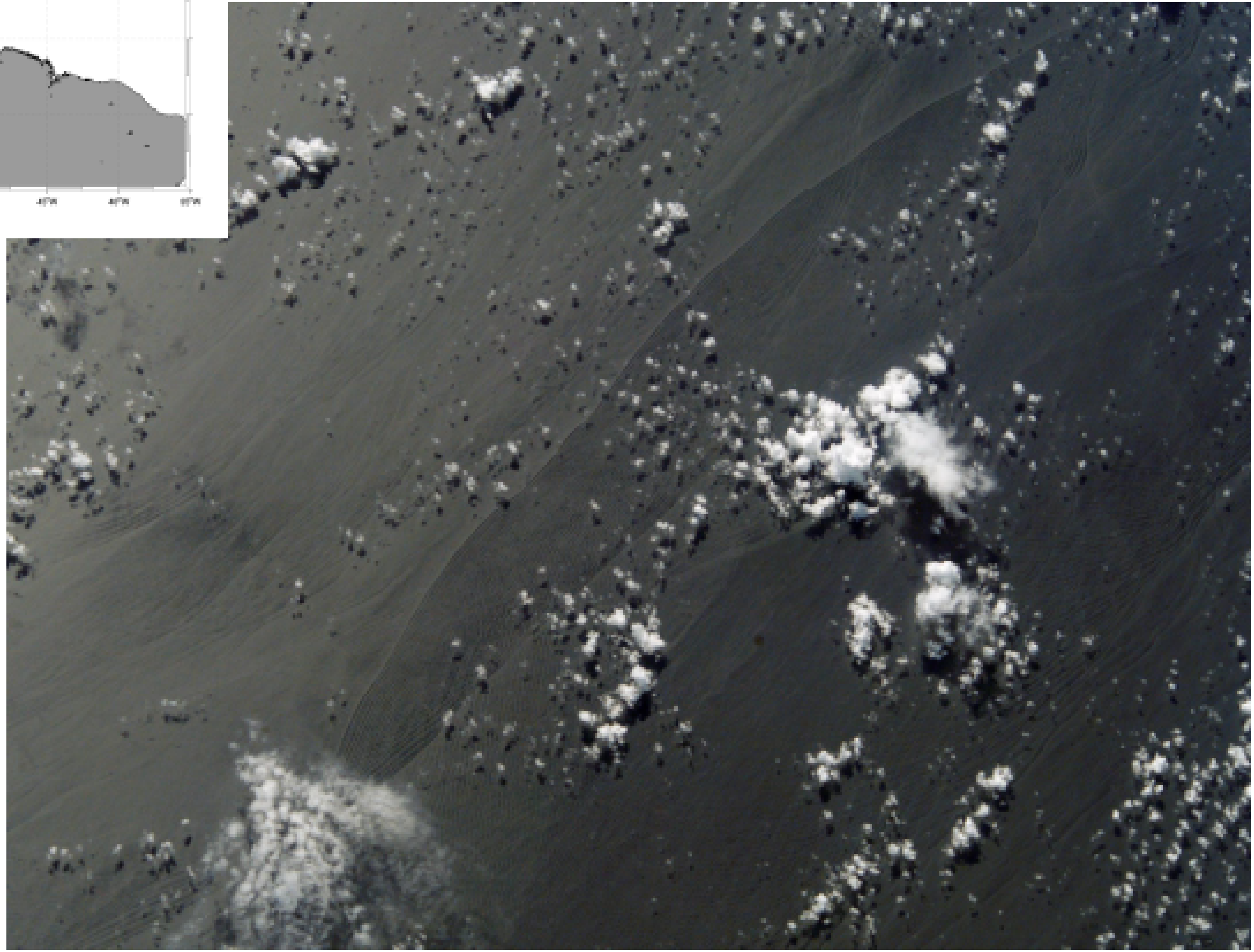


Figure 3. Astronaut photograph (ISS005-E-5322) acquired on 18 June 2002 at 1256 UTC. The image shows internal wave surface signatures similar to those observed in SIR-C SAR image (Figure 4). Orientation and image size unknown. [Image courtesy of Earth Sciences and Image Analysis, NASA-Johnson Space Center (http://eol.jsc.nasa.gov)]



Natural Slicks:

- Currents
- Large-Scale Waves
- Wind Variability

Marina, University

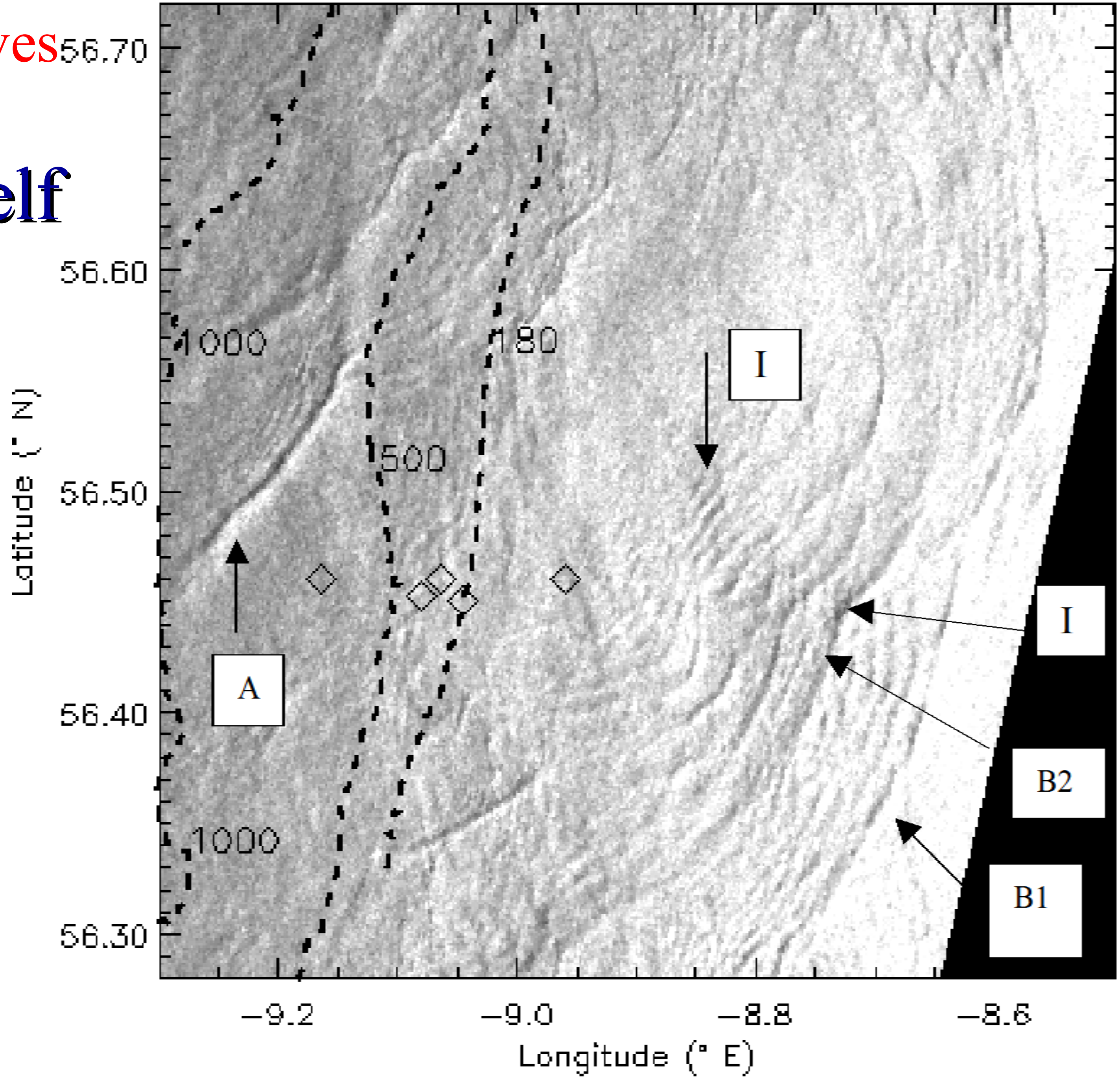

Wind Ripples

We studied the internal wave action on the surfactant film and formation of slick zones on the sea surface. This work is done in frames of ***Project No 1775p*** with Defense Evaluation Research Agency (DERA) UK.

Internal Waves

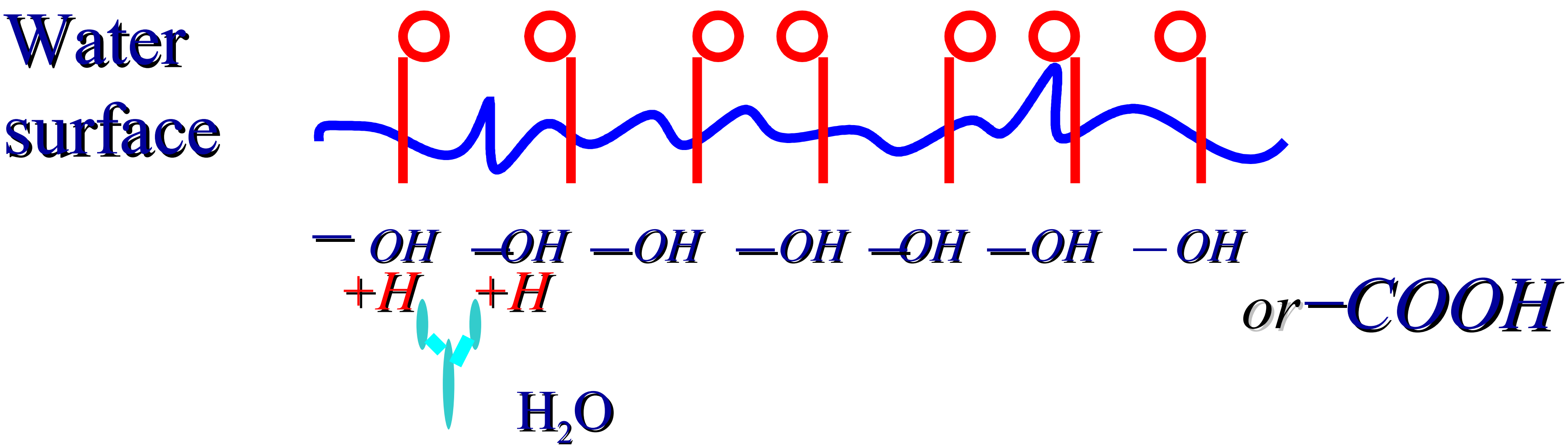
Malin Shelf

SESAME



SURFACTANT MOLECULES

Hydrophobic parts of molecules

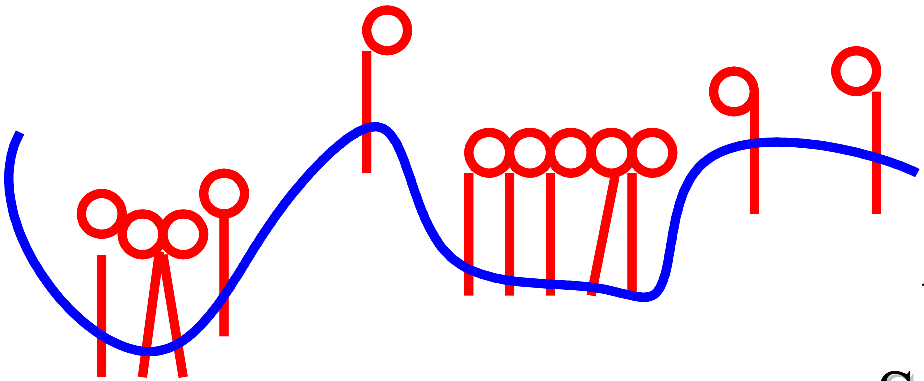


Hydrophilic parts of surfactant molecules

Γ is surface concentration of surfactant

$$\Gamma = \mu/S$$

Film characteristics *Physical*



Surface tension σ

σ_0 = 73 dyn/cm is surface tension of clean water

Surfactant molecules are reduced water surface tension

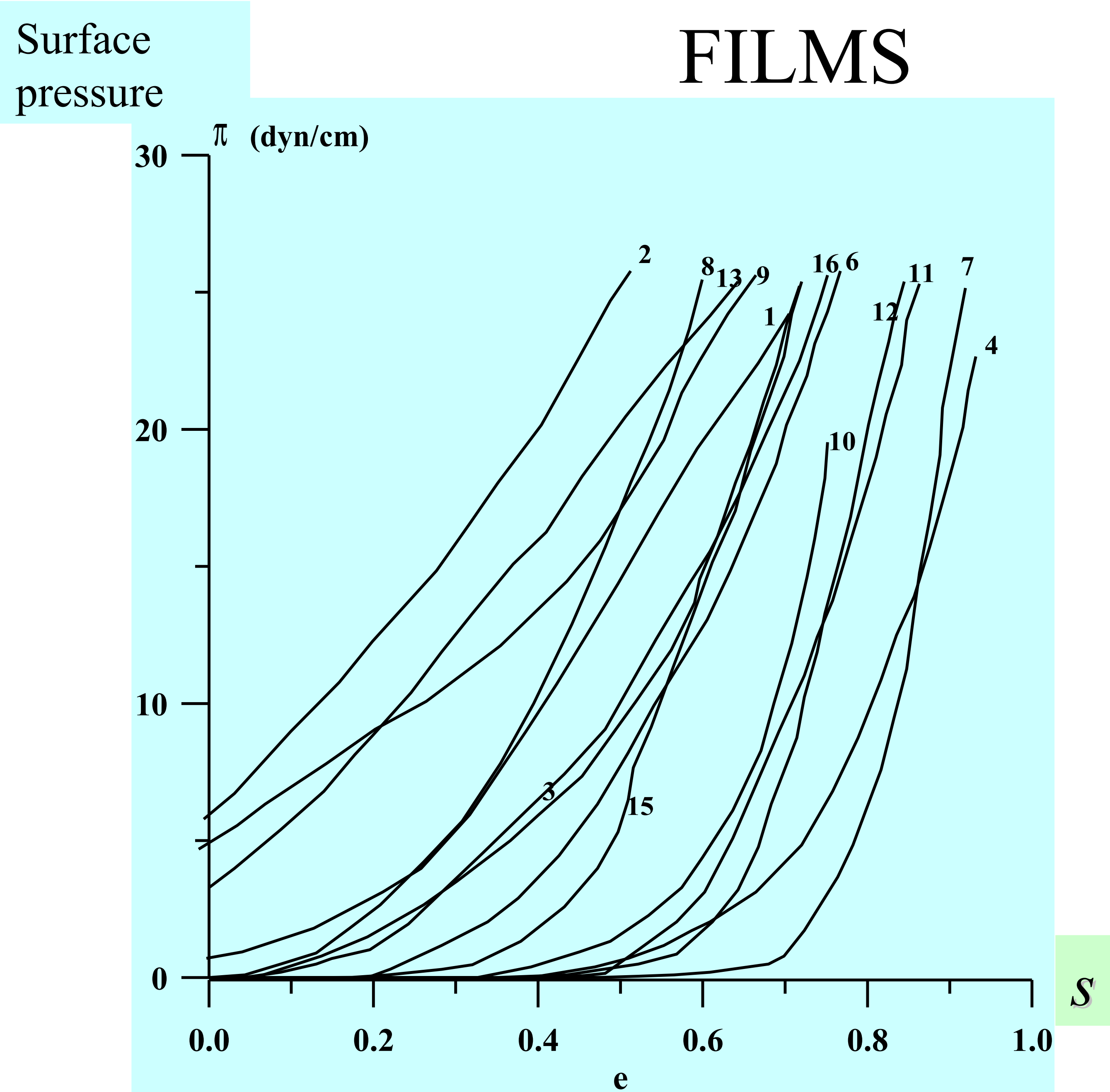
Film pressure $\pi = \sigma_0 - \sigma$

$\pi = \pi(\Gamma)$ is film isotherm

Film elasticity modulus (Young's modulus)

$$E = \Gamma \frac{d\pi}{d\Gamma}$$

ISOTHERMS of SURFACTANT MARINE FILMS



$$\Gamma = \mu/S$$

$$s = \frac{S_0 - S}{S_0} = $$

$$= \frac{\Gamma - \Gamma_0}{\Gamma}$$

Action on the wind ripple

Decrement of capillary-gravity waves

$$\gamma_s = 2\gamma_0 \frac{1 - \dfrac{Ek^2}{\rho\sqrt{2\nu}\,\omega^{3/2}} + \dfrac{E^2k^3}{4\sqrt{2}\,\nu^{3/2}\omega^{5/2}\rho^2}}{1 - 2\dfrac{Ek^2}{\rho\sqrt{2\nu}\,\omega^{3/2}} + 2\left(\dfrac{Ek^2}{\rho\sqrt{2\nu}\,\omega^{3/2}}\right)^2}.$$

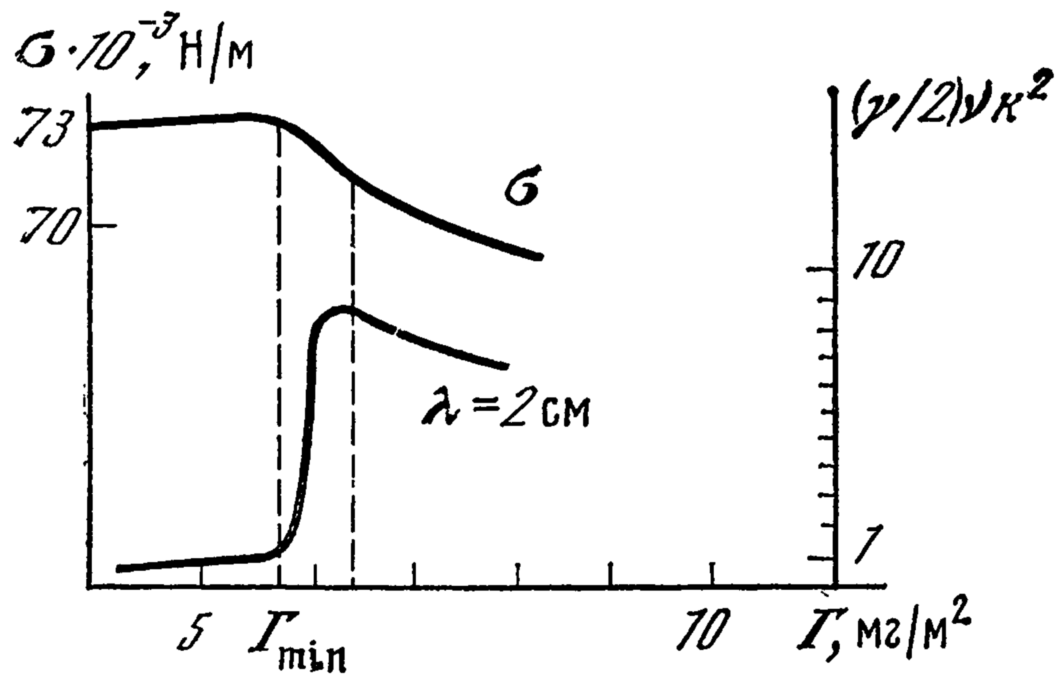
ω is wave frequency

ν is water viscosity

k is wave number

$\gamma_0 = 2\nu k^2$

Small variation of Γ leads to strong variation of decrement, and therefore, visibility of slicks



QUESTIONS

Where is the film concentration Γ increase enough for slick formation?

What does influence on the growth of Γ in situ?

Concentration balance equation

Vector form

$$\frac{\partial C}{\partial t} + \vec{V}\nabla C = D\Delta C + I$$

Scalar form

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + I$$

There is basic equation for any kind of pollution

In atmosphere (aerosol)

In ocean (soluble substances, salt, insoluble fine-dyspersated substances, surfactant films)

In rivers (oil, man-made pollutions)

Heat “pollution”

Basic Model:

2D Advection-Diffusion-Relaxation Equation

$$\frac{\partial \Gamma}{\partial t} + div(\vec{u}\Gamma) = D\Delta\Gamma + \frac{\Gamma_0 - \Gamma}{\tau} + Q$$

u(x,y,t) – surface current

D – horizontal diffusion

τ- relaxation (exchange with deeper layers)

1D Advection Model

$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}(u\Gamma) = 0$$

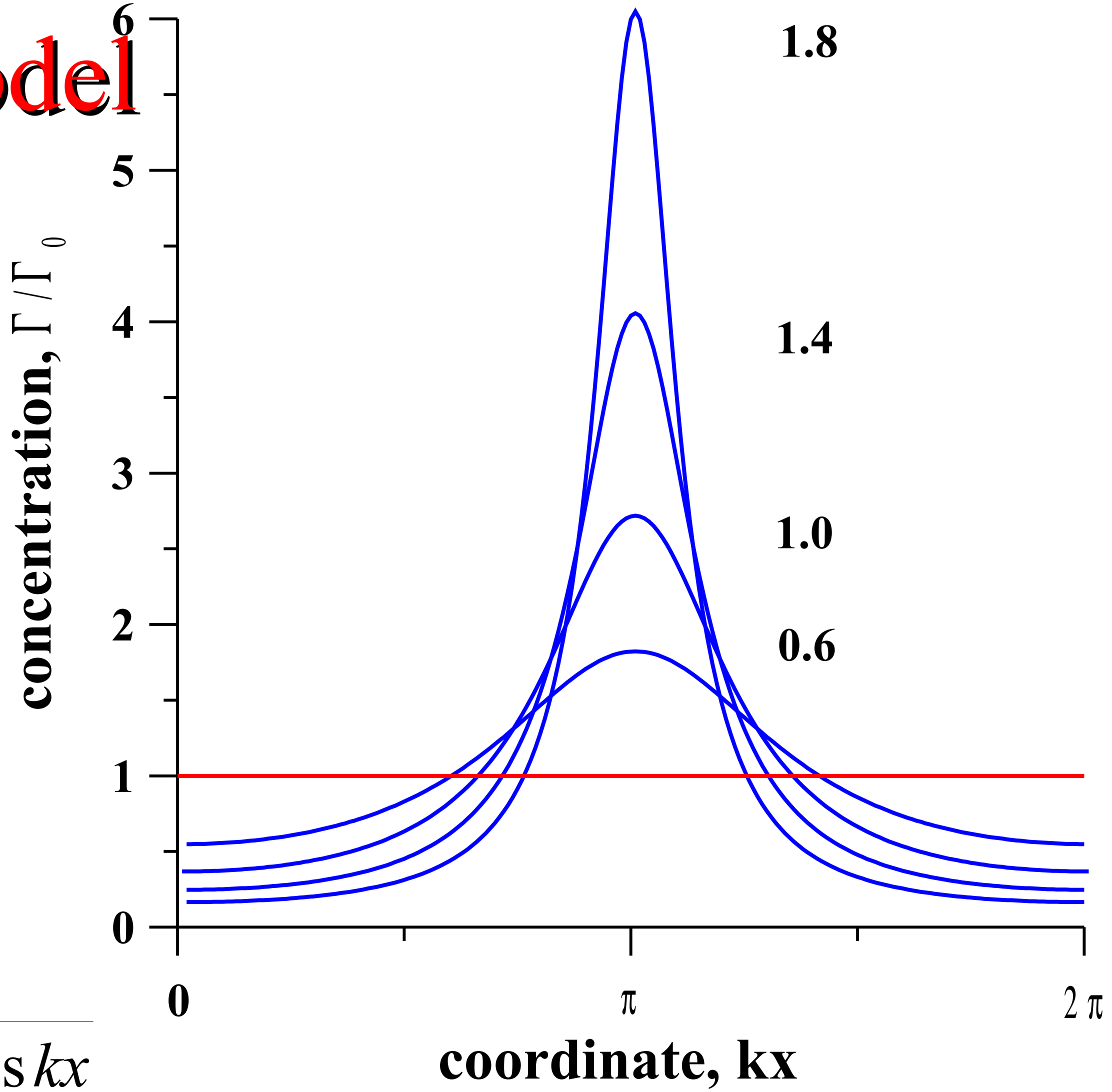
$$u(x) = u_0 \sin kx$$

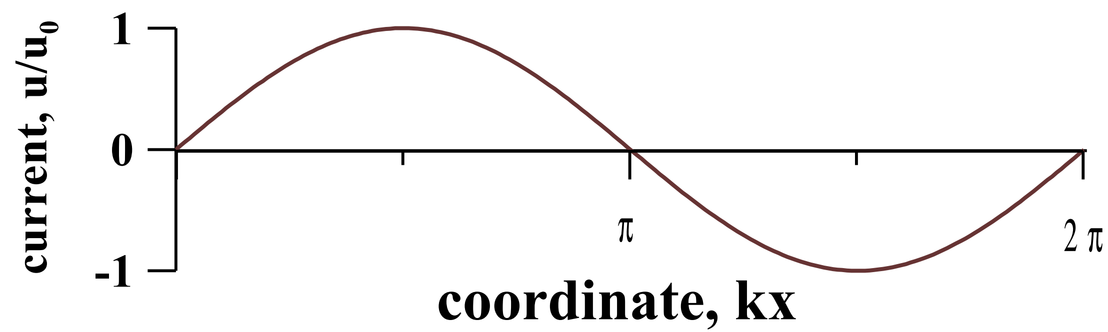
Langmuir Cells:

u ~ 2cm/s,   L ~ 100 m

$$\Gamma(x,t) = \frac{\Gamma_0}{\cosh ku_0 t + \sinh ku_0 t \cos kx}$$

T ~ 13 min

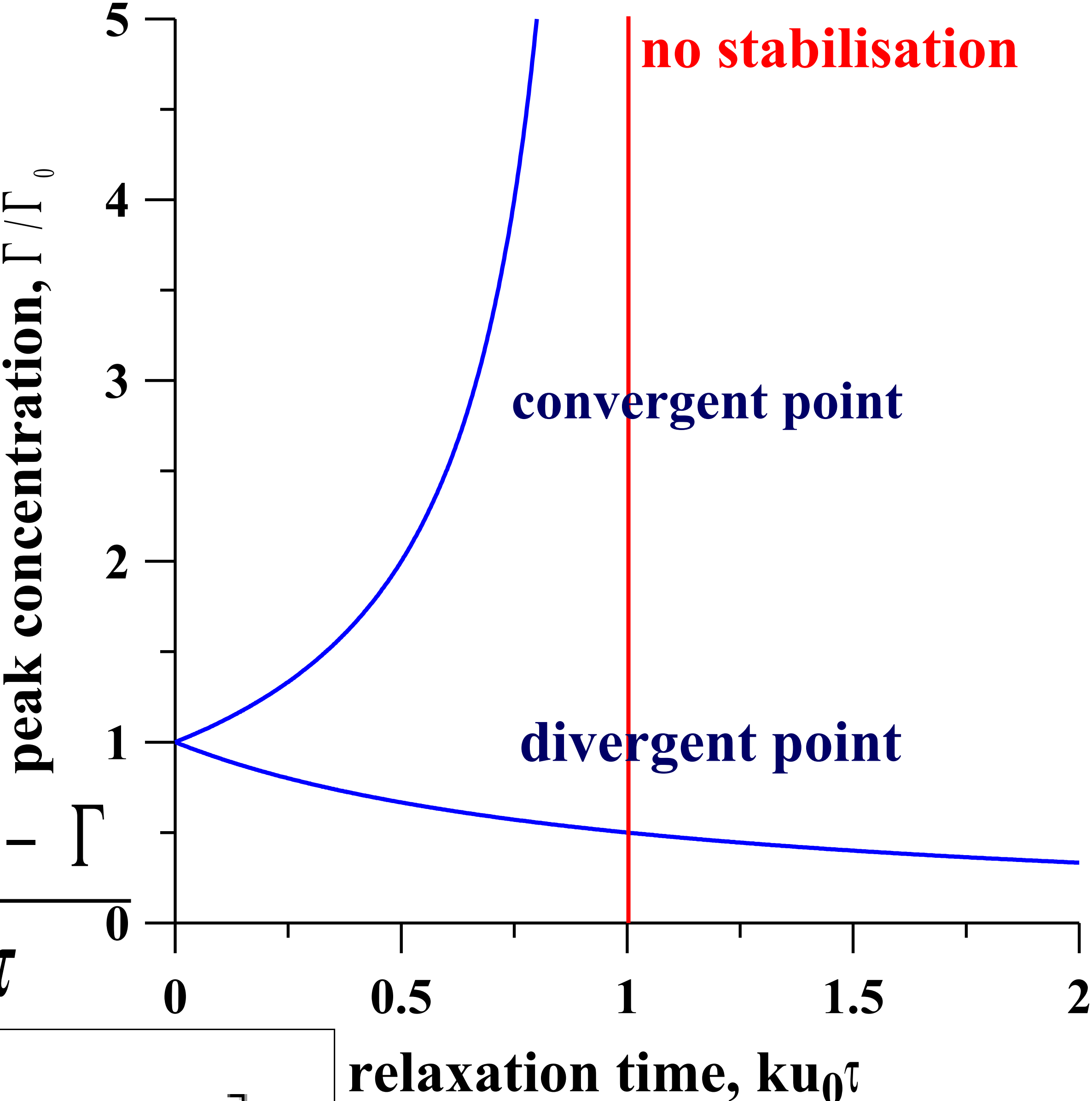




1D Advection-Relaxation Model

$$\frac{\Gamma_{div}^{\infty}}{\Gamma_0} = \frac{1}{1 + ku_0\tau}.$$

$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}(u\Gamma) = \frac{\Gamma_0 - \Gamma}{\tau}$$



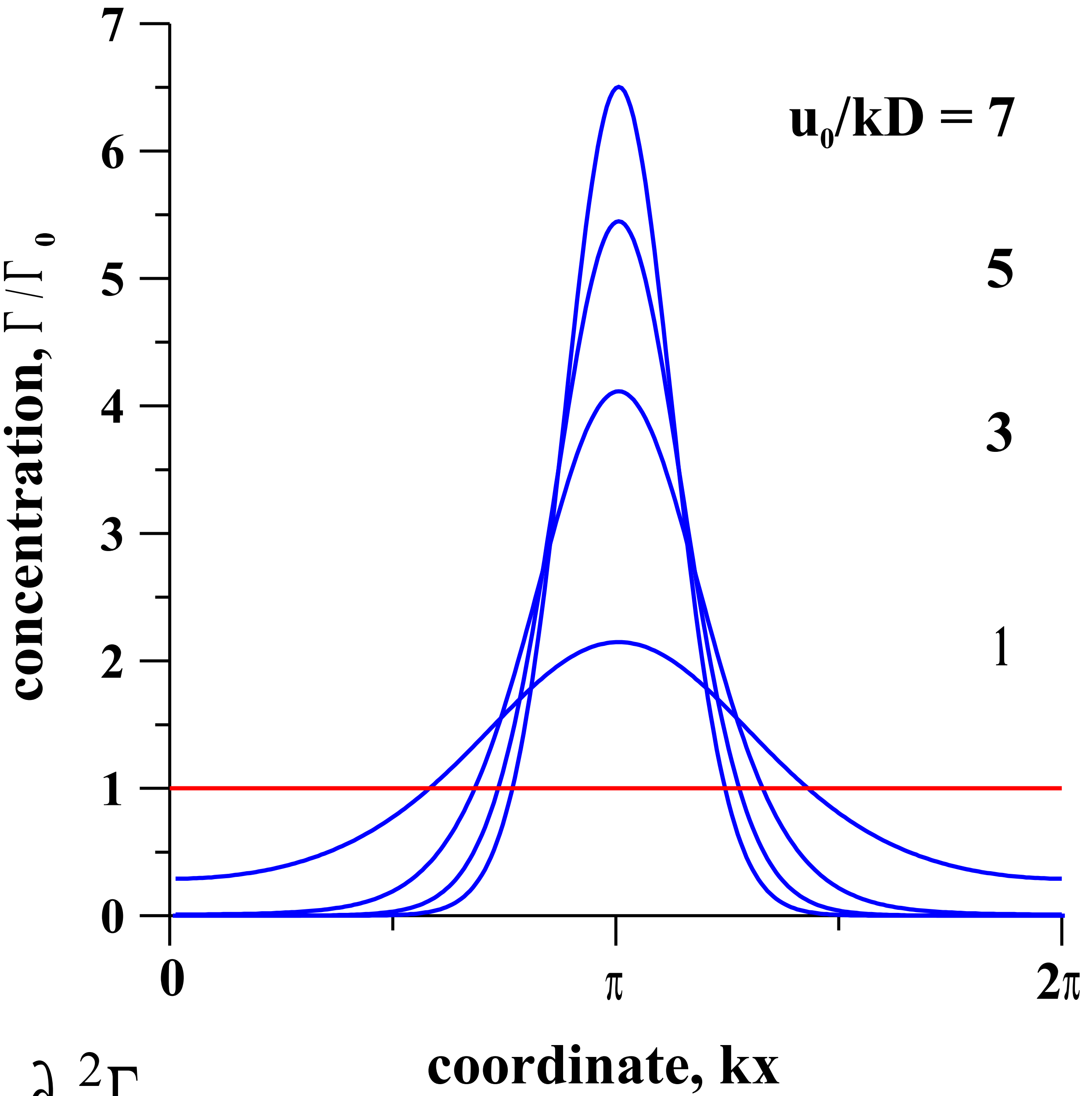
$$\frac{\Gamma_{con}}{\Gamma_0} = \frac{1}{1 - ku_0\tau}\left[1 - ku_0\tau \exp\left((ku_0\tau - 1)\frac{t}{\tau}\right)\right].$$

1D Advection – Diffusion Model

I_0 is the modified Bessel function of zero order

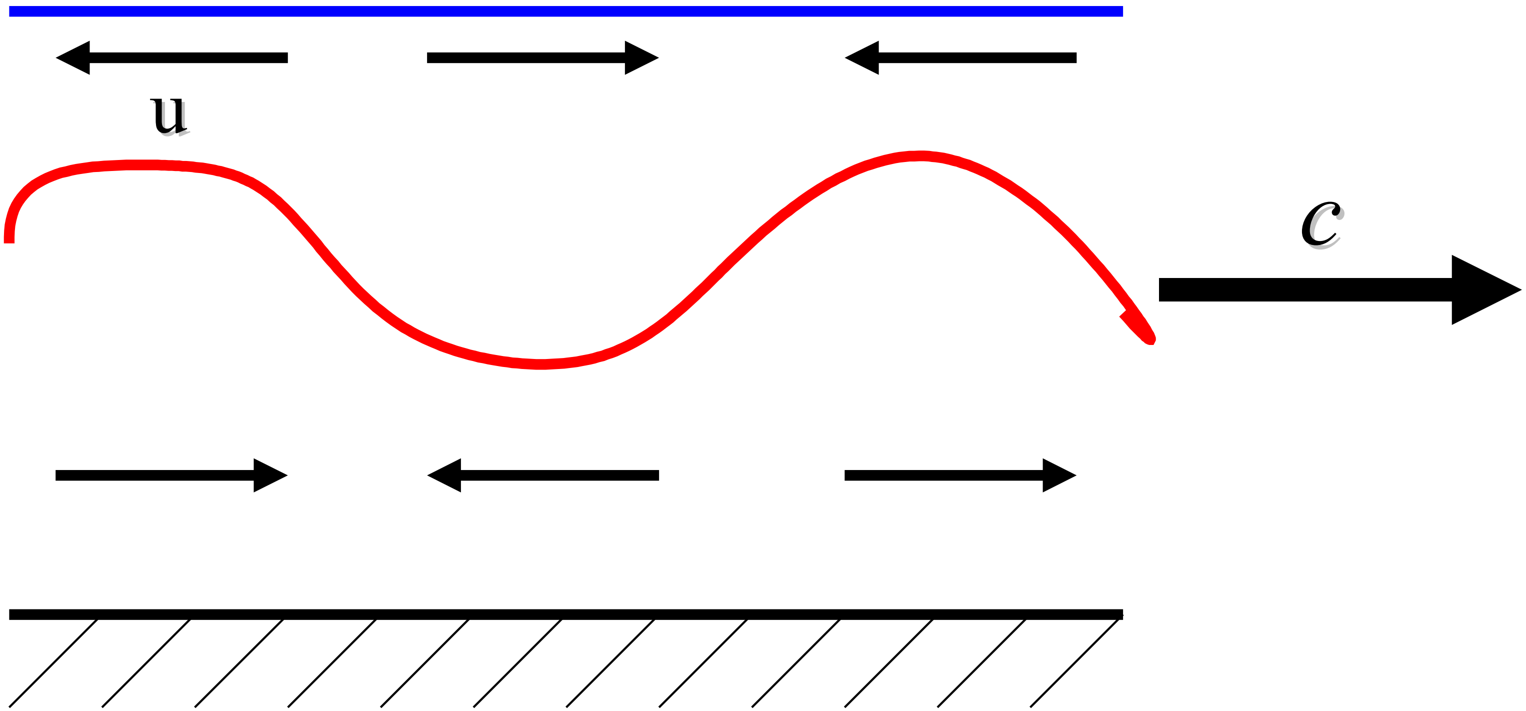
Steady solution

$$\frac{\Gamma(x)}{\Gamma_0} = \frac{\exp\left(-\frac{u_0 \cos kx}{kD}\right)}{I_0\left(\frac{u_0}{kD}\right)}$$



$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}(u\Gamma) = D\frac{\partial^2 \Gamma}{\partial x^2}$$

Internal Wave Velocity Field



$$u(x,t) = u(x - ct) \qquad c > u$$

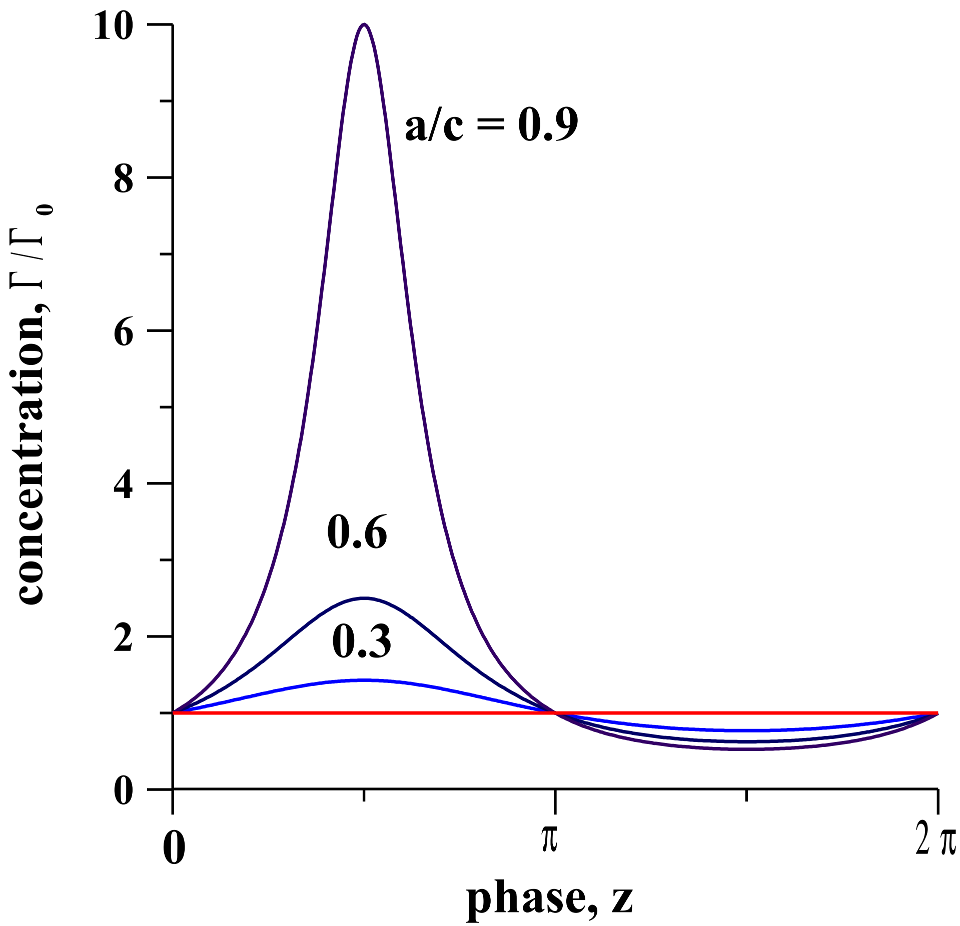
$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}\left(u(x - ct)\Gamma\right) = 0$$

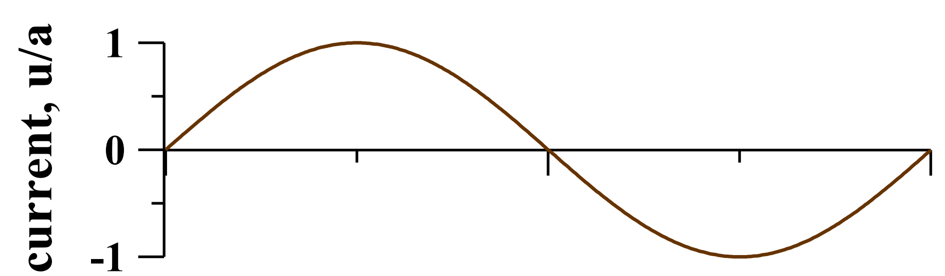
1D Advection Model for Wave Disturbance

$$u(x,t) = u(x - ct)$$

Steady – State Solution

$$\frac{\Gamma}{\Gamma_0} = \frac{c}{c - u}$$





Do not satisfy mean-level condition for concentration

Exact Unsteady Solution

$$\frac{\Gamma(z,t)}{\Gamma_0} = \frac{1-\varepsilon^2}{\Pi}$$

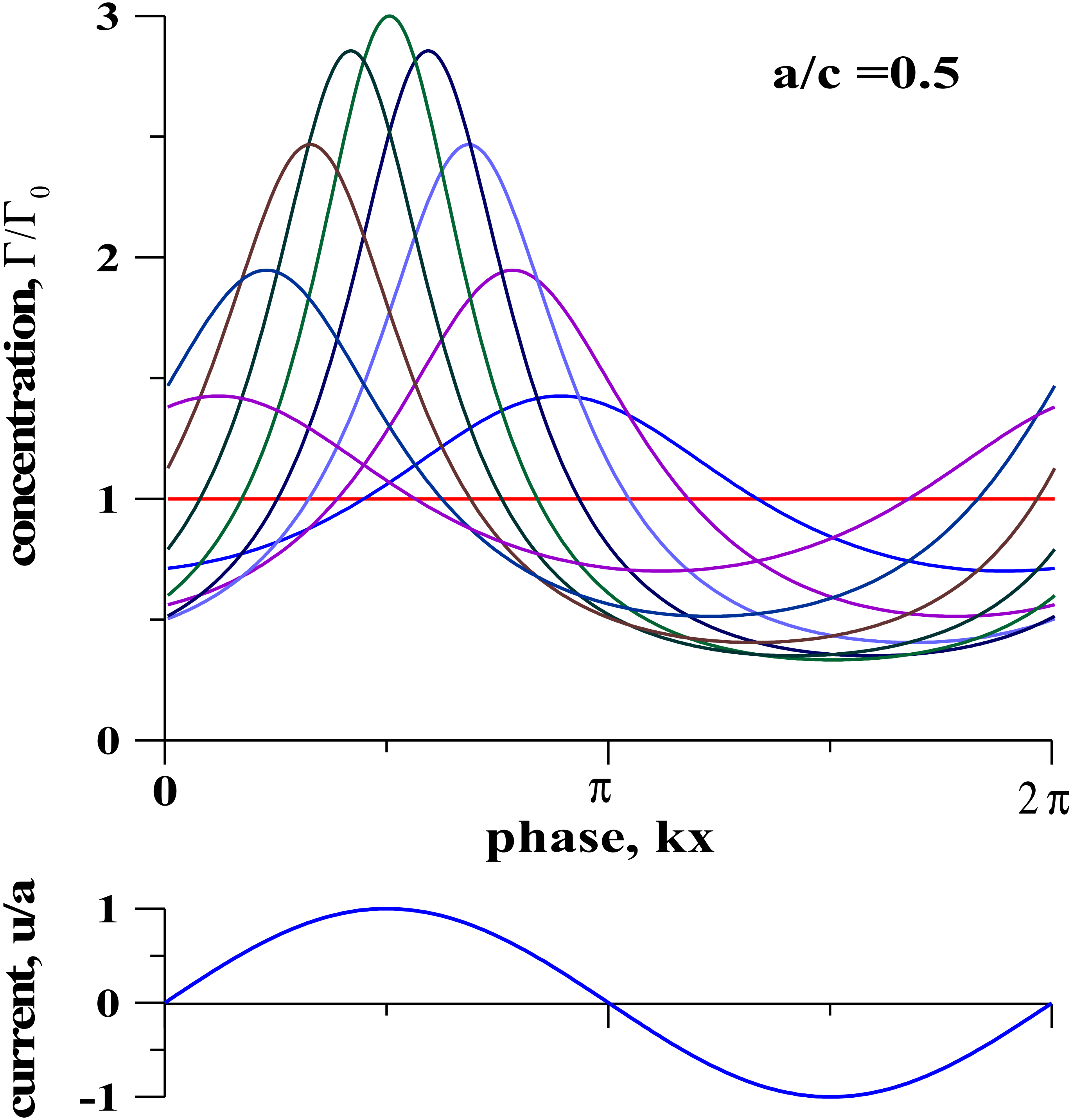
$$\Pi = 1-\varepsilon\left(1-\cos\Omega t\right)\sin kz + \varepsilon\sqrt{1-\varepsilon^2}\sin\Omega t\cos kz - \varepsilon^2\cos\Omega t$$

$$\varepsilon = \frac{a}{c} \qquad \Omega = \sqrt{1-\varepsilon^2}\,kc$$

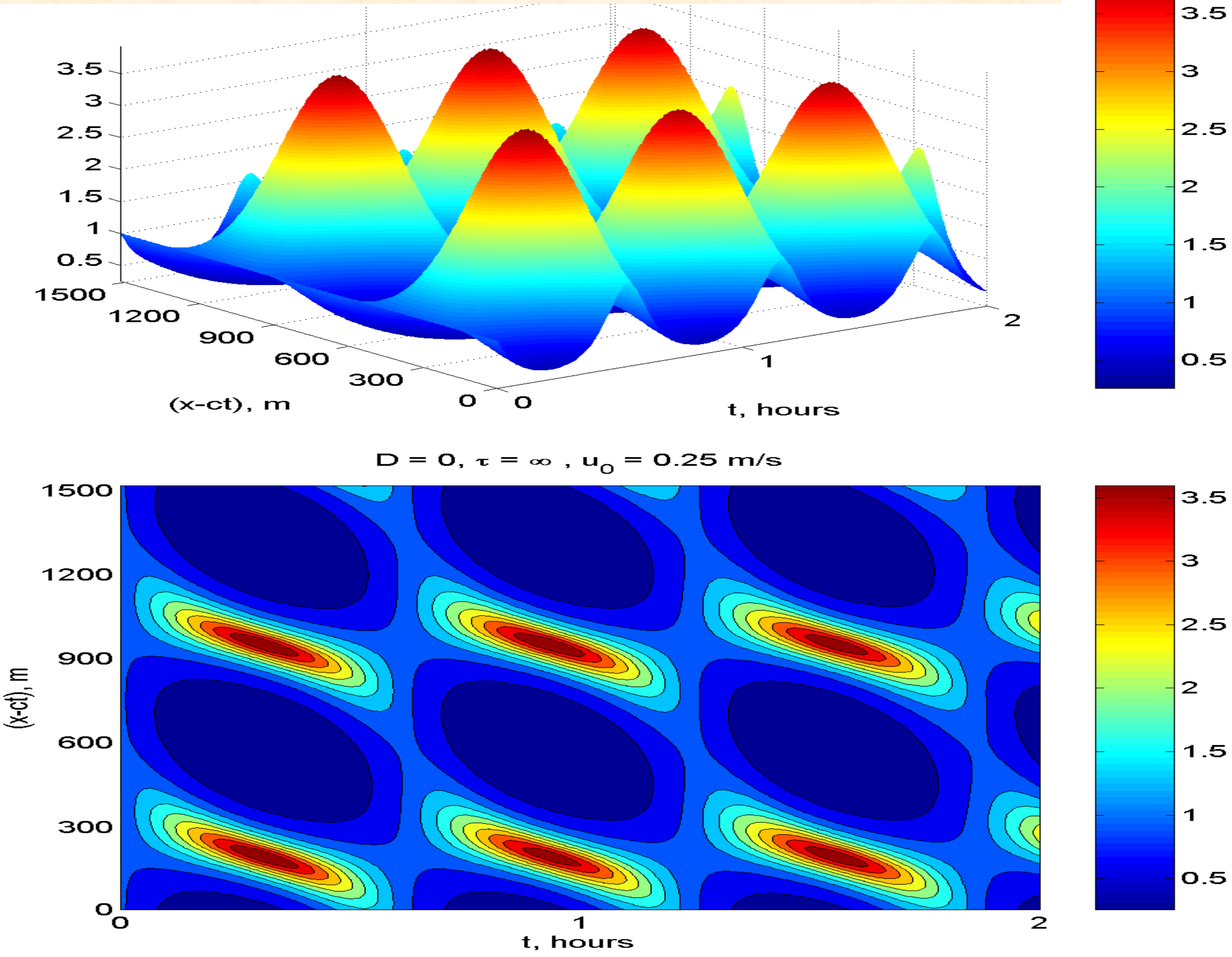
$$u(x,t) = a\sin kz, \qquad z = x - ct$$

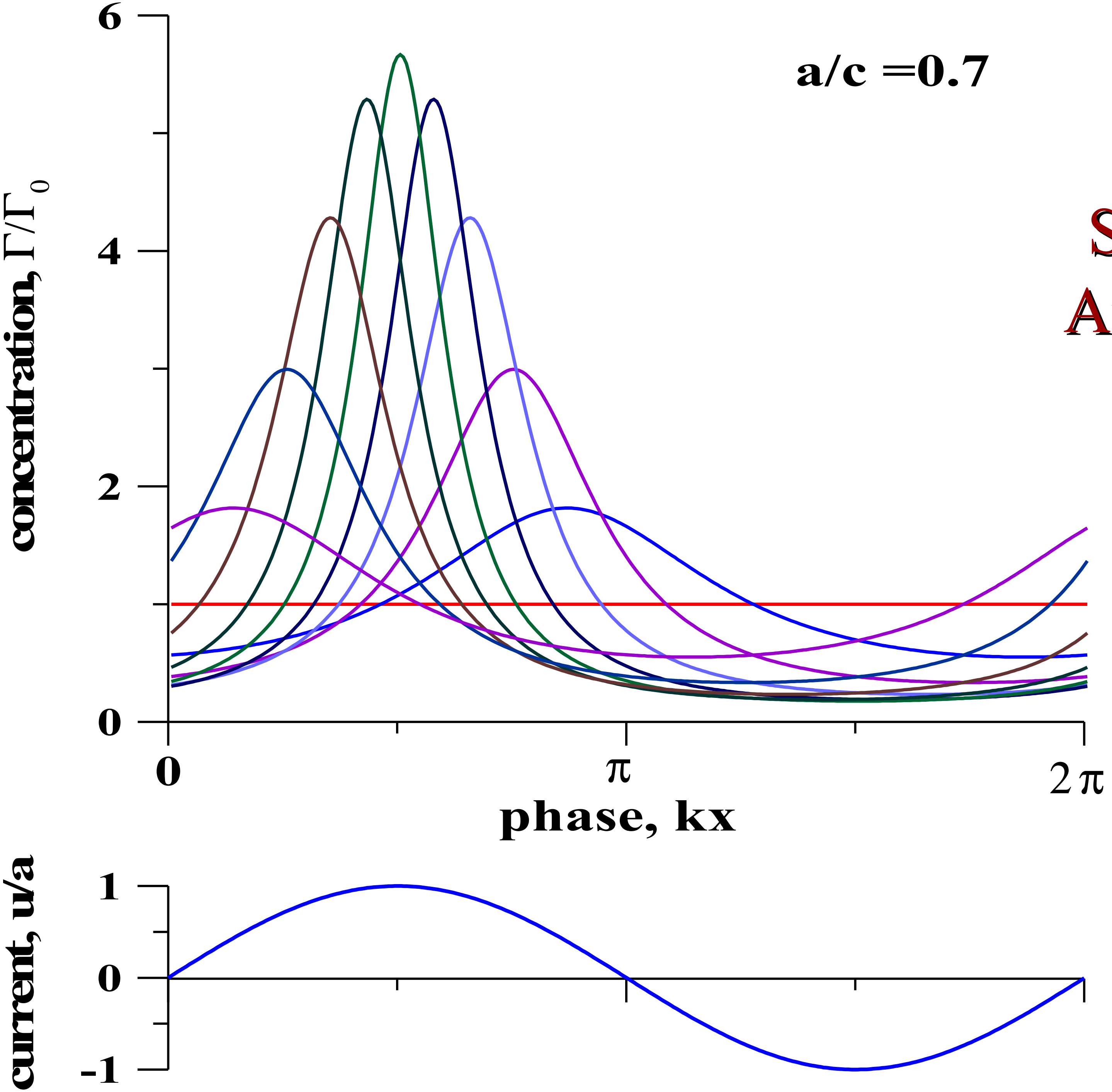
Steady-state Amplification

2



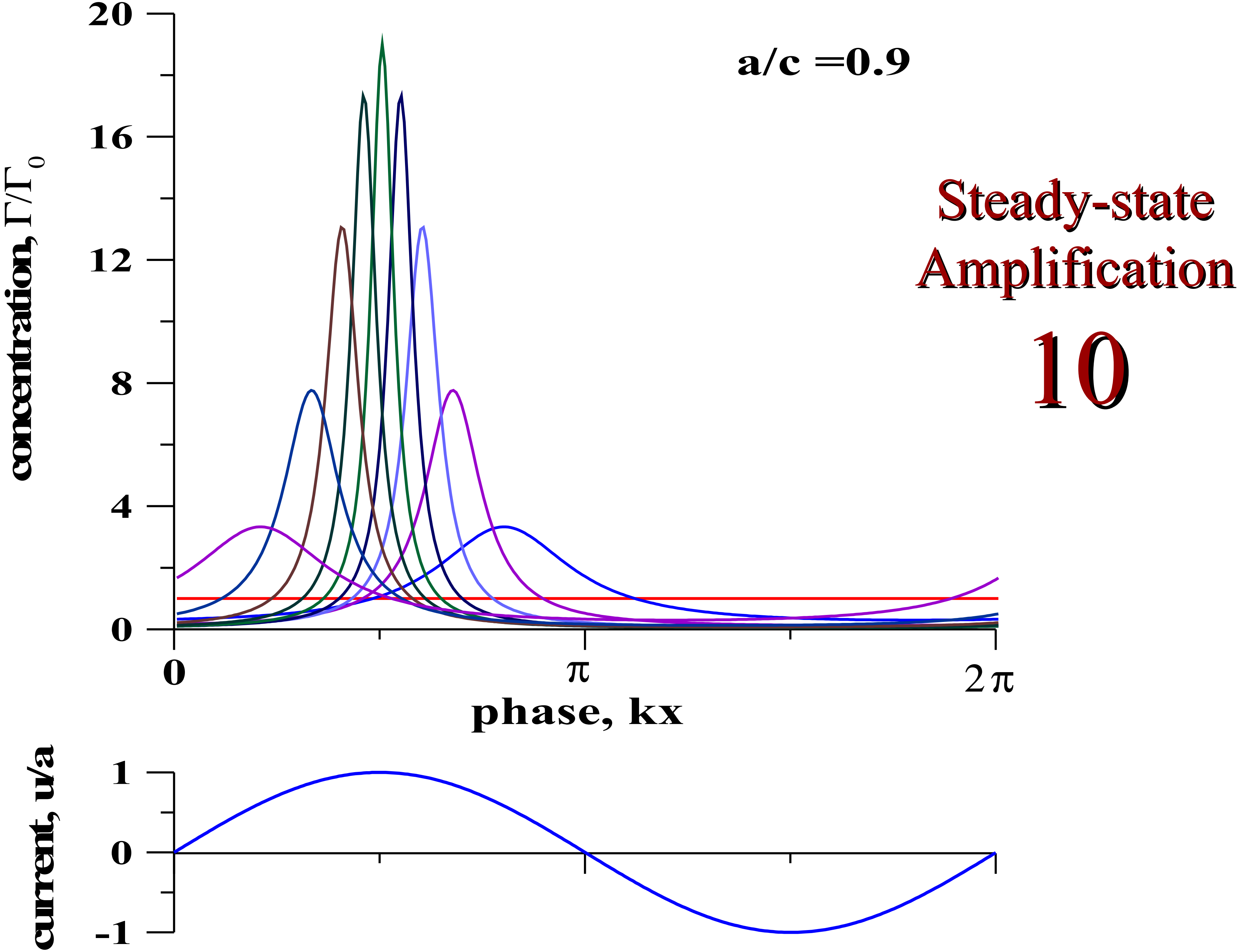
Surfactant variation in field of sine wave (ε= 0.6)



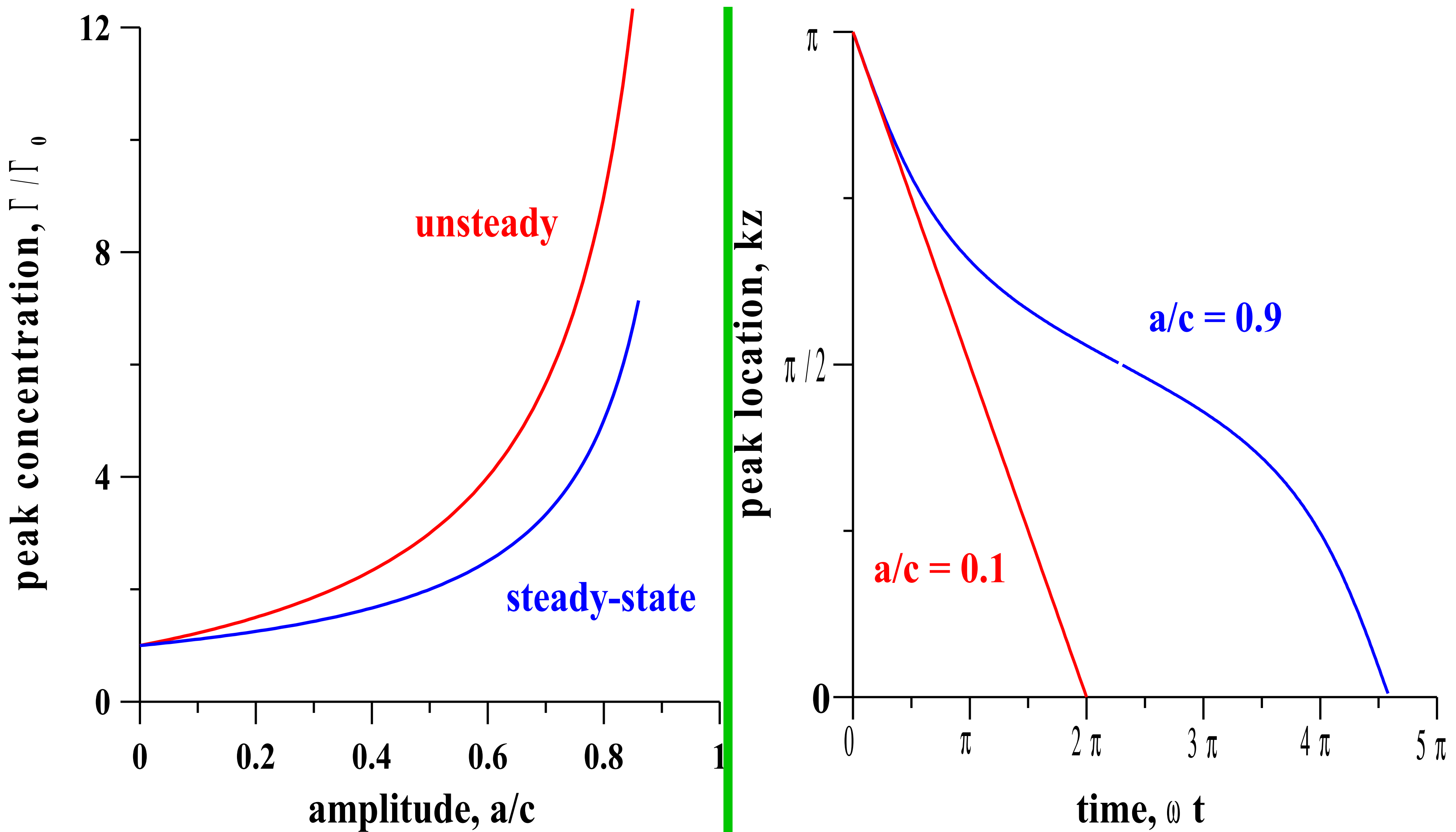


Steady-state Amplification

3.3


20
16
12
8
4
0
concentration, Γ/Γ0
a/c =0.9
Steady-state
Amplification
10
0
π
2π
phase, kx
current, u/a
1
0
-1

Unsteady Solution



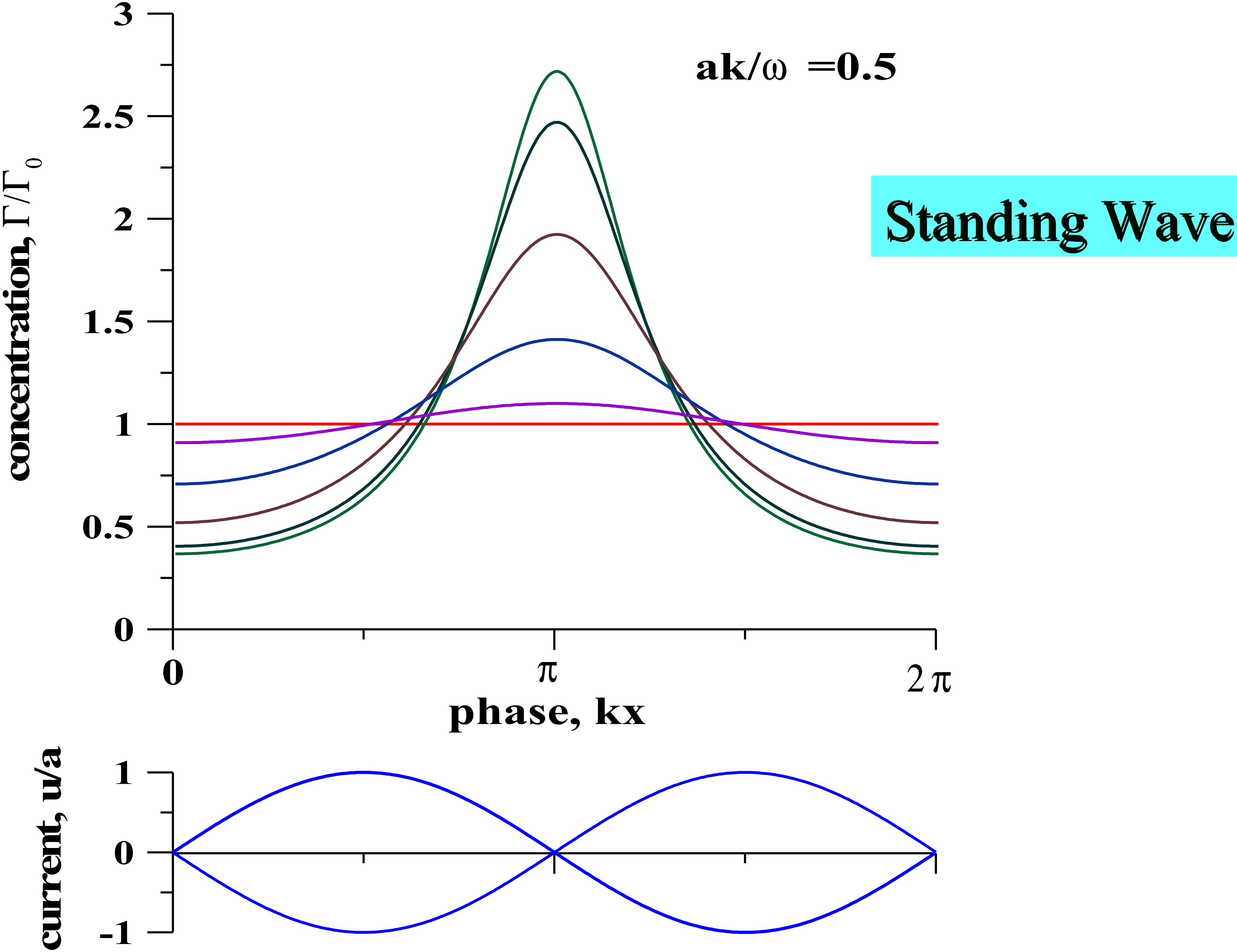
Standing Waves

$$u = a \sin kx \sin \omega t$$

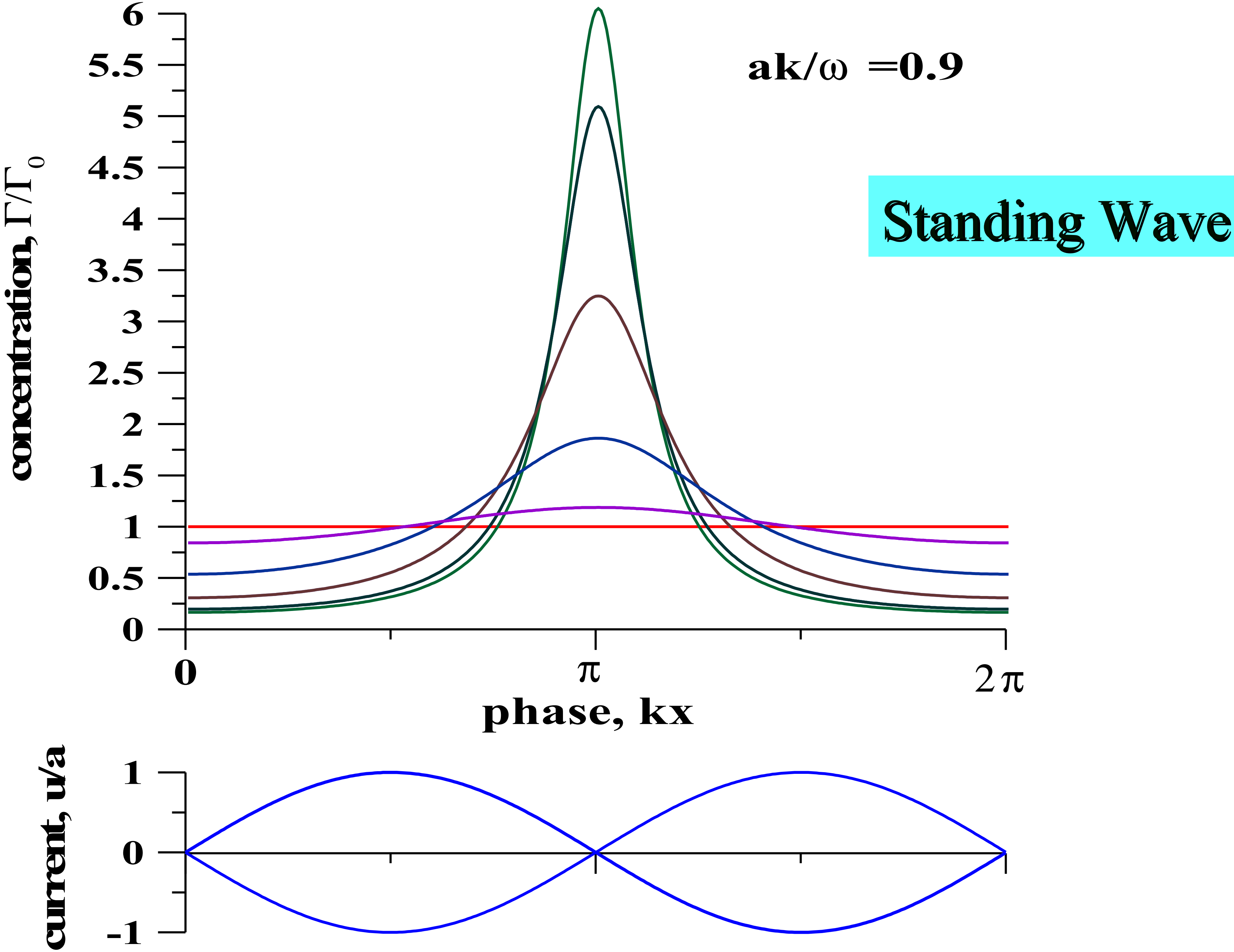
$$\frac{\Gamma(x,t)}{\Gamma_0} = \frac{1}{\cosh\left(2\delta \sin^2(\omega t/2)\right) + \cos kx \sinh\left(2\delta \sin^2(\omega t/2)\right)}$$

$$\delta = ak/\omega$$

$$\frac{\Gamma_{peak}}{\Gamma_0} = \exp\left(2\delta \sin^2(\omega t/2)\right), \qquad (kx = \pi)$$



ak/ω =0.5
Standing Wave
concentration, Γ/Γ0
3
2.5
2
1.5
1
0.5
0
0
π
2π
phase, kx
current, u/a
1
0
-1


ak/ω =0.9
Standing Wave
concentration, Γ/Γ0
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
π
2π
phase, kx
current, u/a
1
0
-1

Influence of diffusion and relaxation

$$\frac{\partial \Gamma}{\partial t}+\frac{\partial}{\partial x}\left(u(x,t)\Gamma\right)=D\frac{\partial^2 \Gamma}{\partial x^2}+\frac{\Gamma_0-\Gamma}{\tau}.$$

$$\Gamma=\Gamma_0\left(1+G(x,t)\right), \qquad G<<1,$$

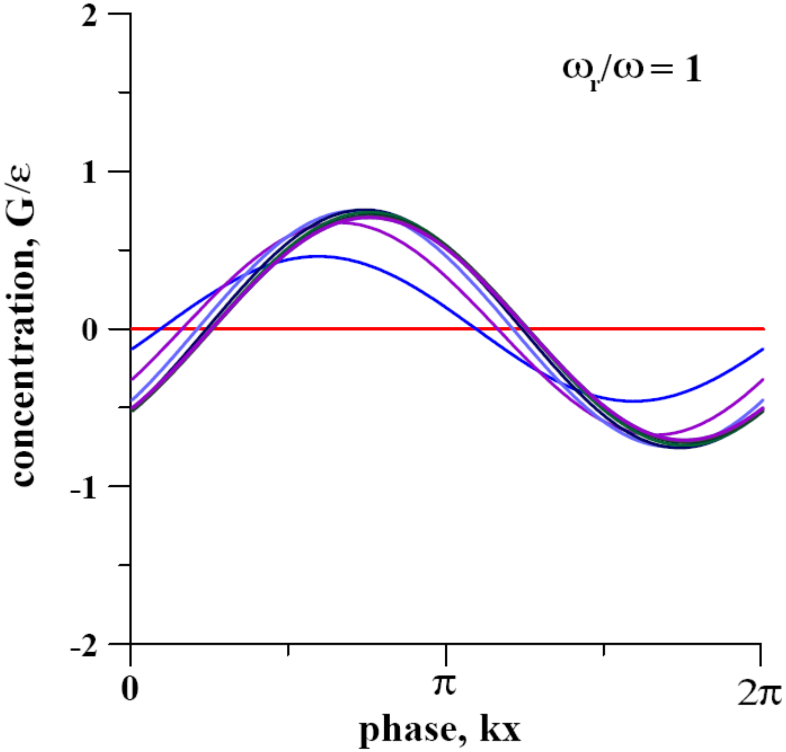
$$\frac{\partial G}{\partial t}+\frac{G}{\tau}-D\frac{\partial^2 G}{\partial x^2}=-\frac{\partial u(x,t)}{\partial x}.$$

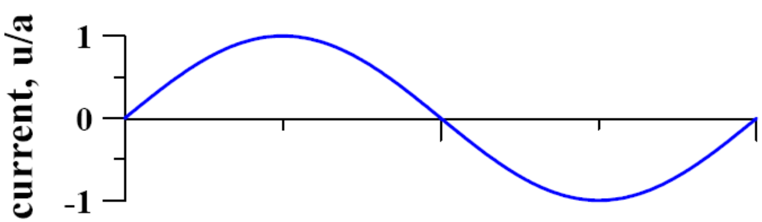
$$G(x,t)=-\int_0^t\int_{-\infty}^{+\infty}\frac{\partial u(x',t')}{\partial x'}\frac{1}{\sqrt{4\pi D(t-t')}}\exp\left[-\frac{t-t'}{\tau}-\frac{(x-x')^2}{4D(t-t')}\right]dt'dx'.$$

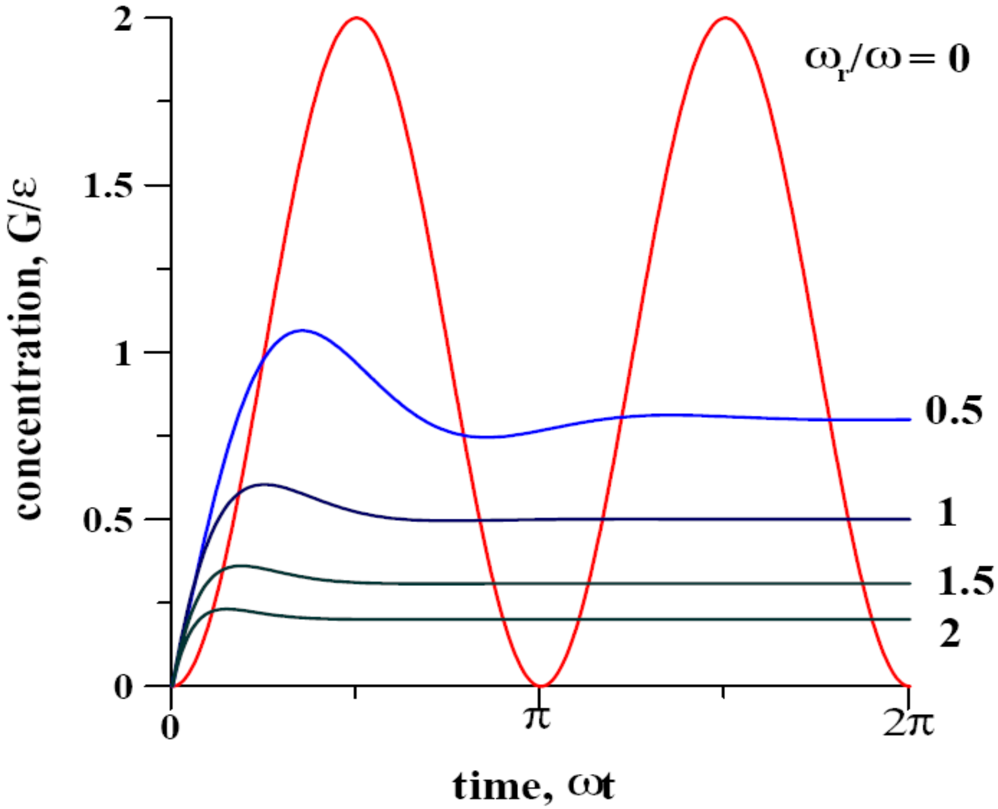
$$G(x,t)=\frac{\varepsilon}{1+\omega_r^2/\omega^2}\left\{\left[\frac{\omega_r}{\omega}\cos(kz-\omega t)-\sin(kz-\omega t)\right]\exp(-\omega_r t)-\frac{\omega_r}{\omega}\cos kz+\sin kz\right\}$$

$$G(z)=\frac{\varepsilon}{\sqrt{1+\omega_r^2/\omega^2}}\sin(kz-\theta),\qquad \theta=\mathrm{atan}(\omega_p/\omega).\ \varepsilon = a/c$$

$$\omega_r=\frac{1}{\tau}+Dk^2,\qquad z = x\text{-}ct$$



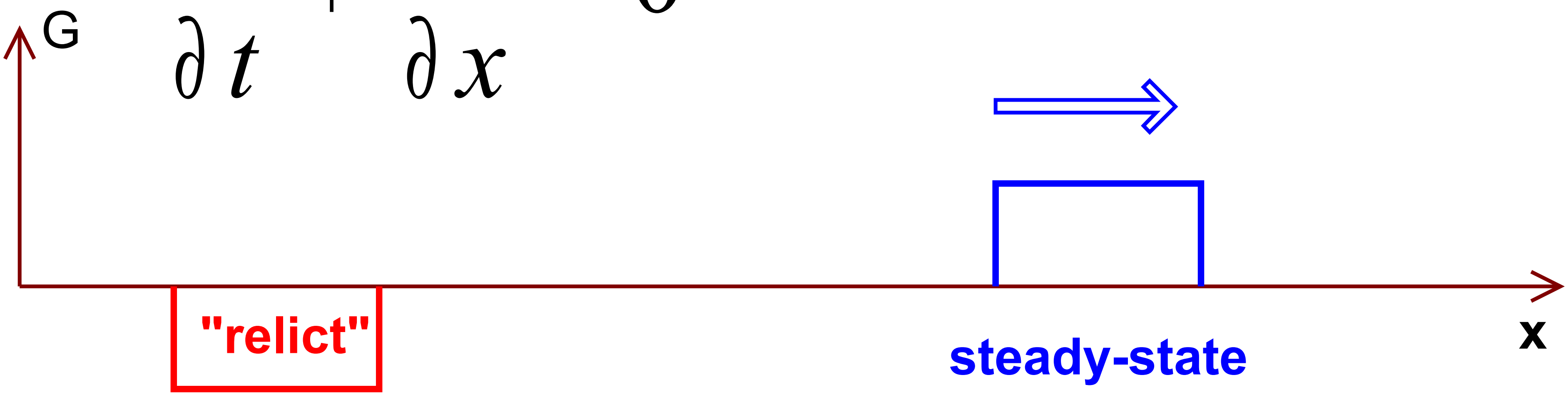




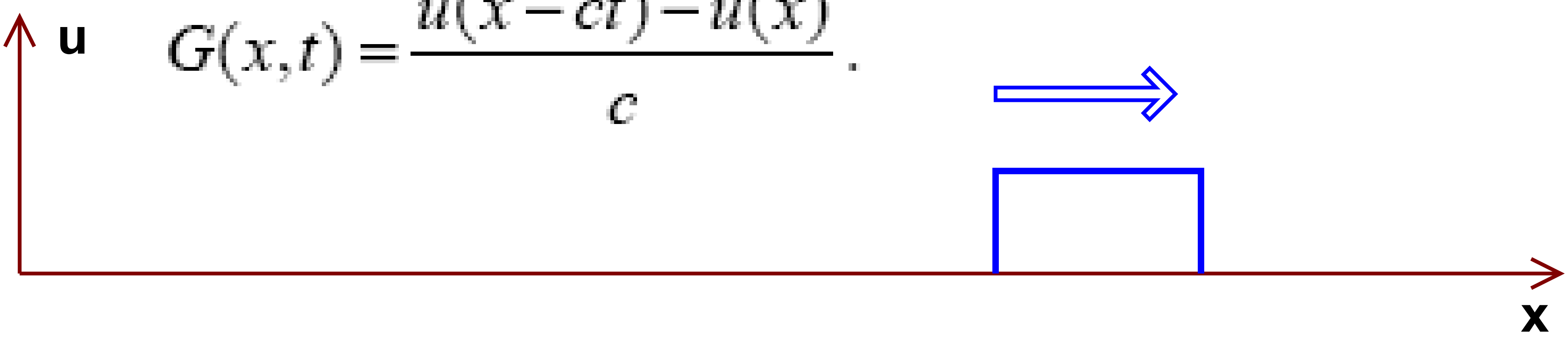
Finite Wave Disturbance

$$\Gamma = \Gamma_0 \left(1 + G(x,t)\right)$$

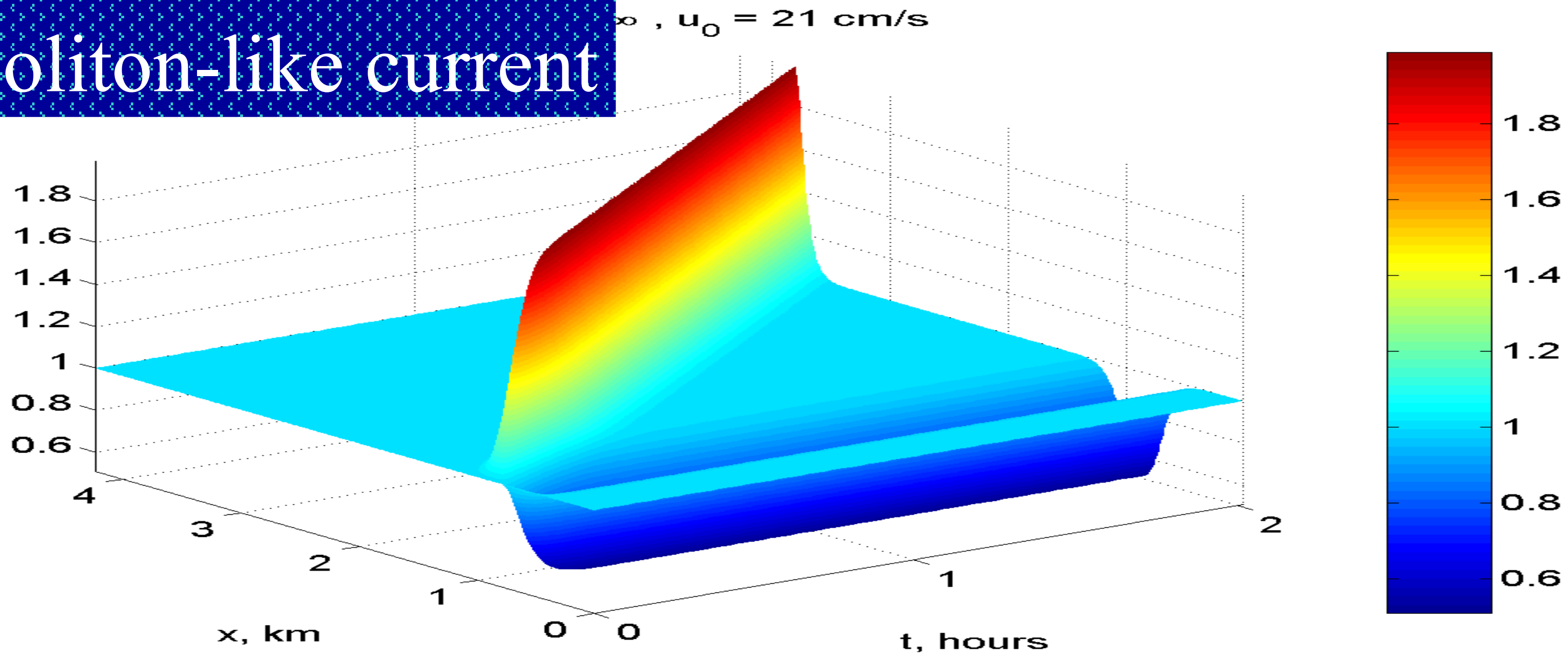
$$\frac{\partial G}{\partial t} + \frac{\partial u}{\partial x} = 0$$

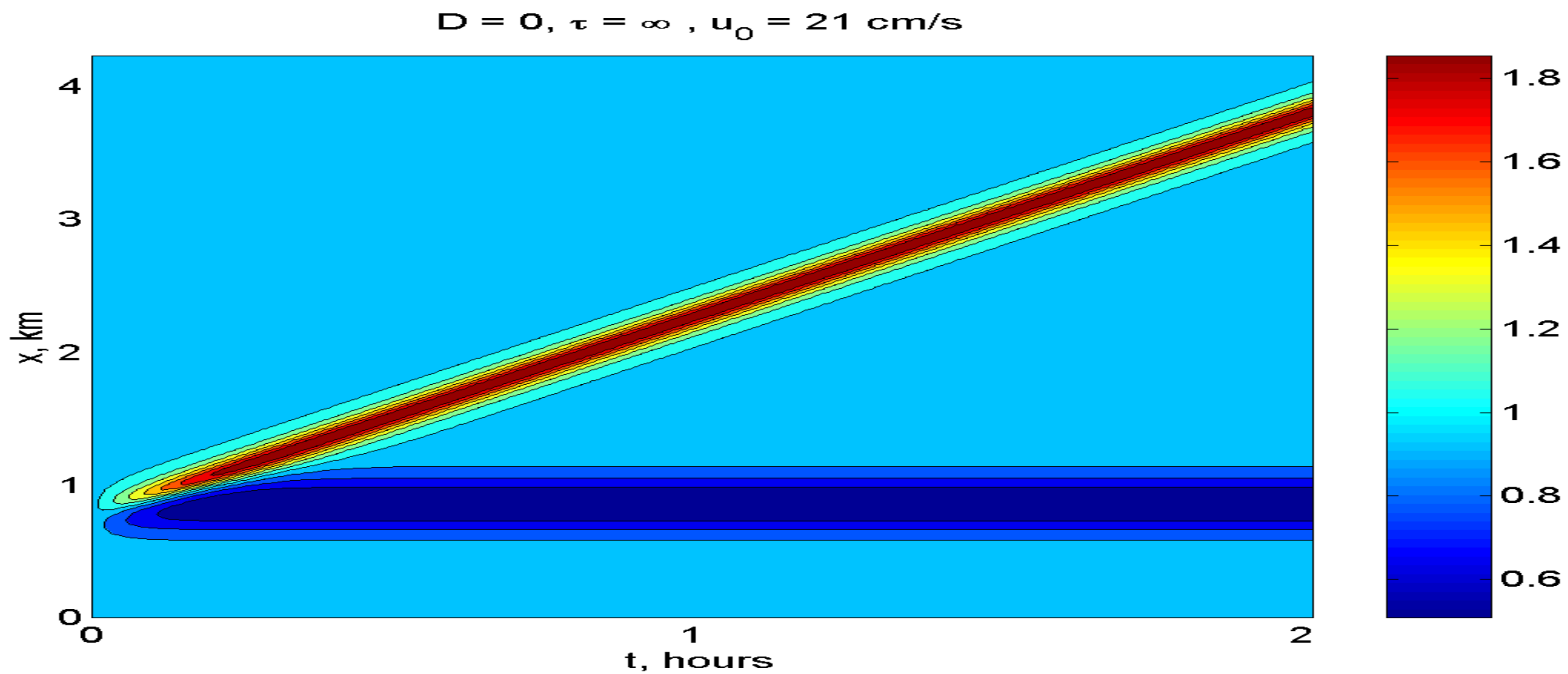


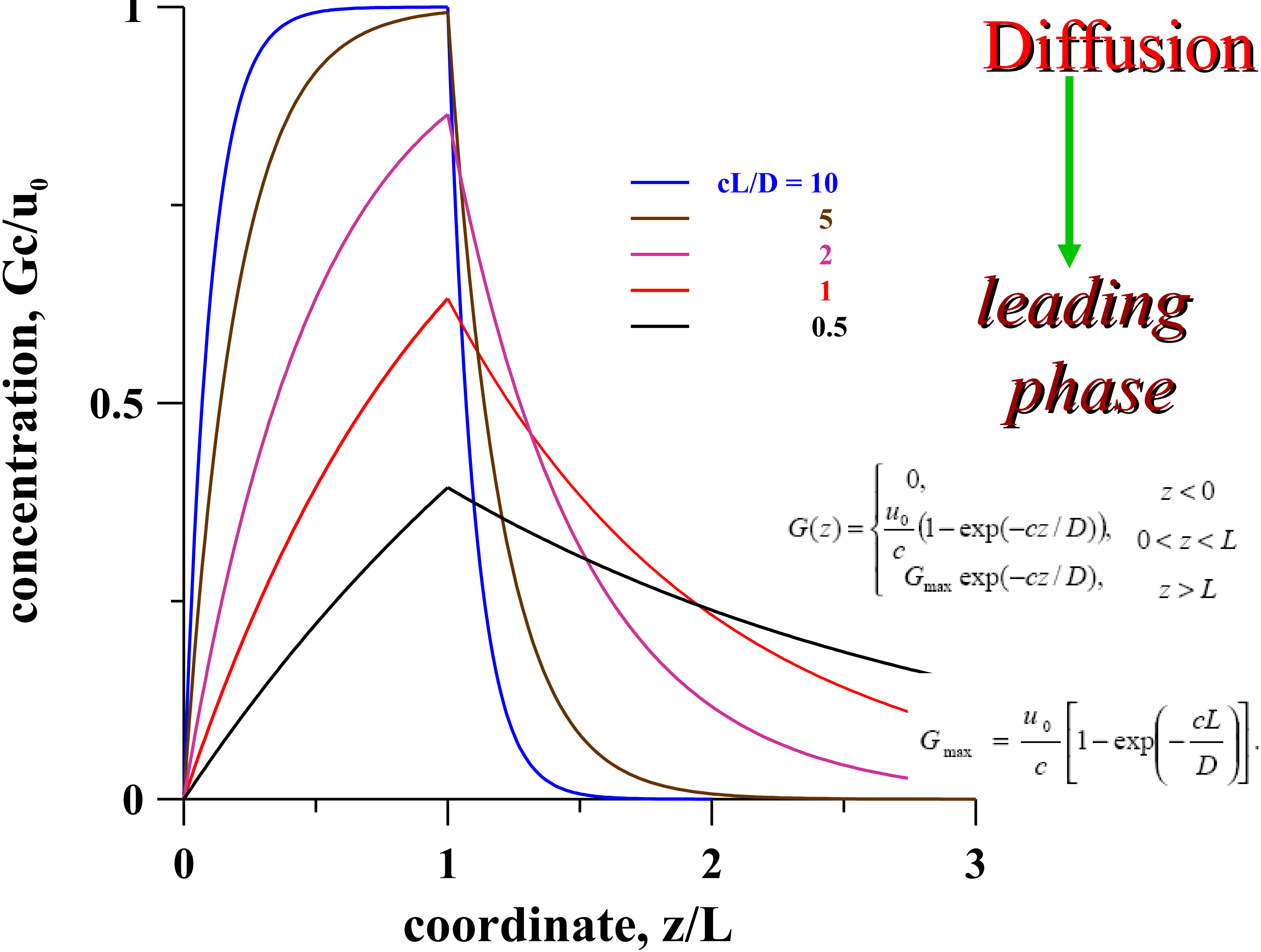
$$G(x,t) = \frac{u(x-ct) - u(x)}{c}.$$



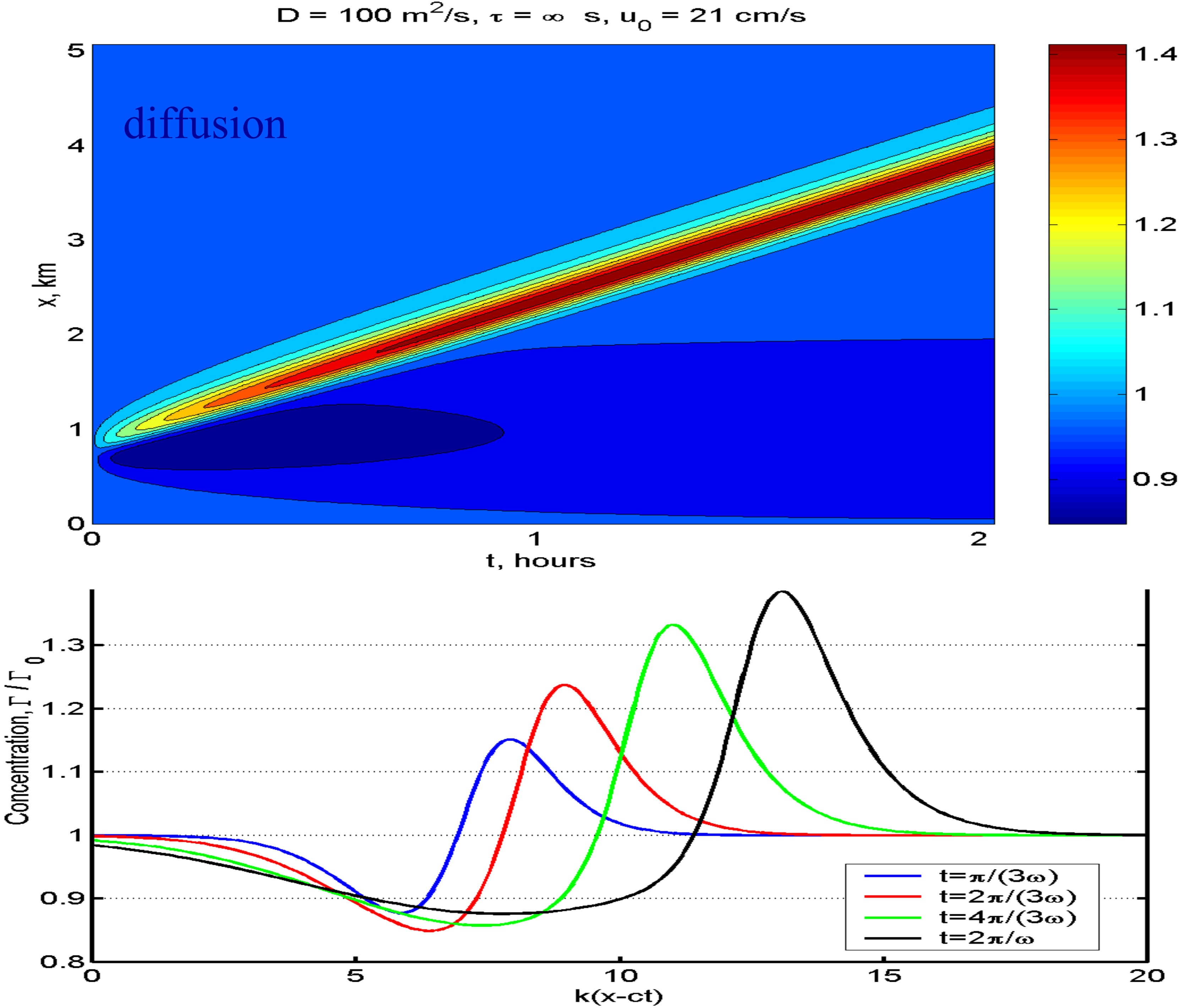
Soliton-like current



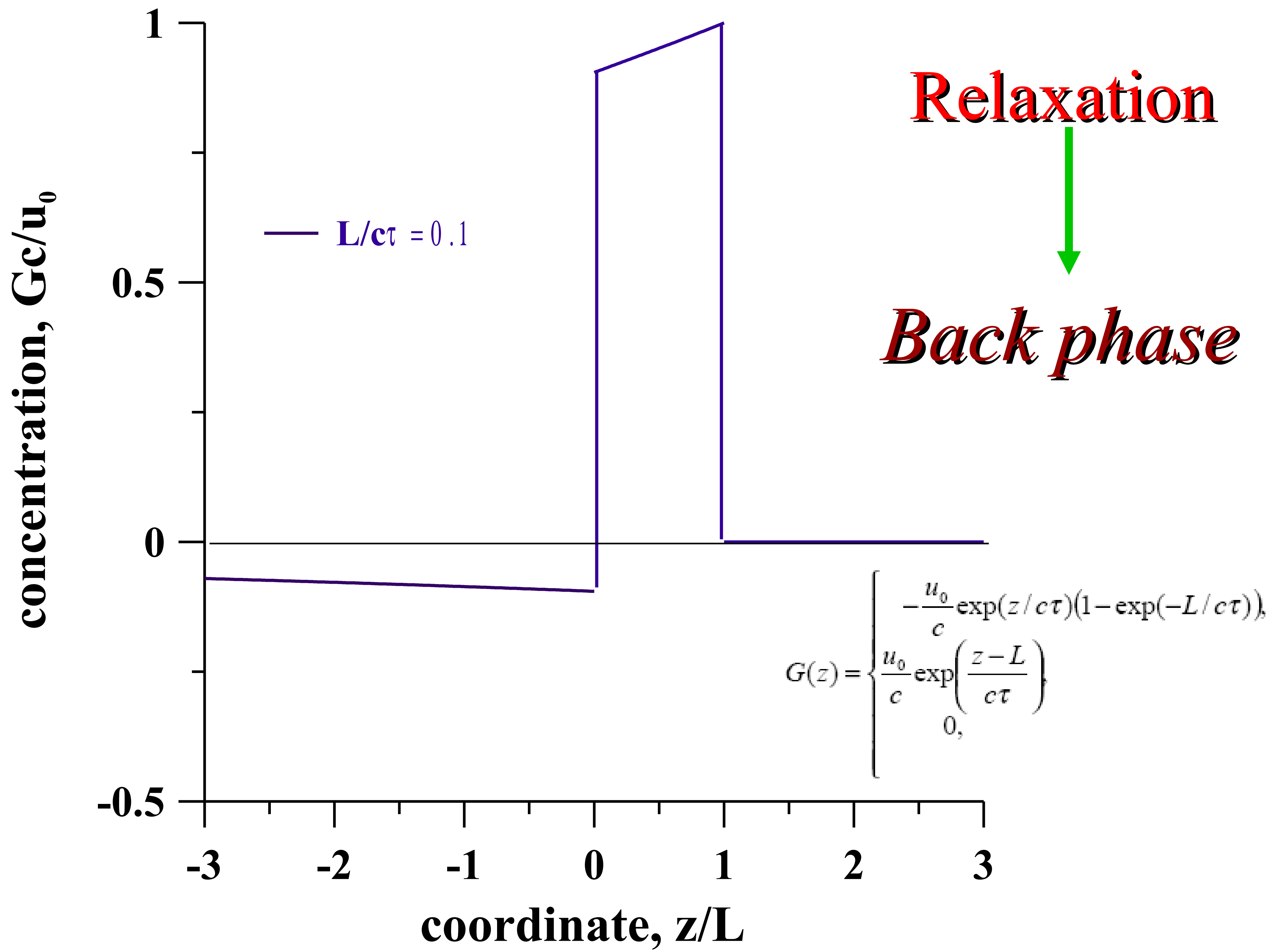




Diffusion
leading
phase
cL/D = 10
5
2
1
0.5
concentration, Gc/u_0
coordinate, z/L
0
0.5
1
0
1
2
3
$$G(z)=\begin{cases}0, & z<0\\ \frac{u_0}{c}\left(1-\exp(-cz/D)\right), & 0<z<L\\ G_{max}\exp(-cz/D), & z>L\end{cases}$$
$$G_{max}=\frac{u_0}{c}\left[1-\exp\left(-\frac{cL}{D}\right)\right].$$

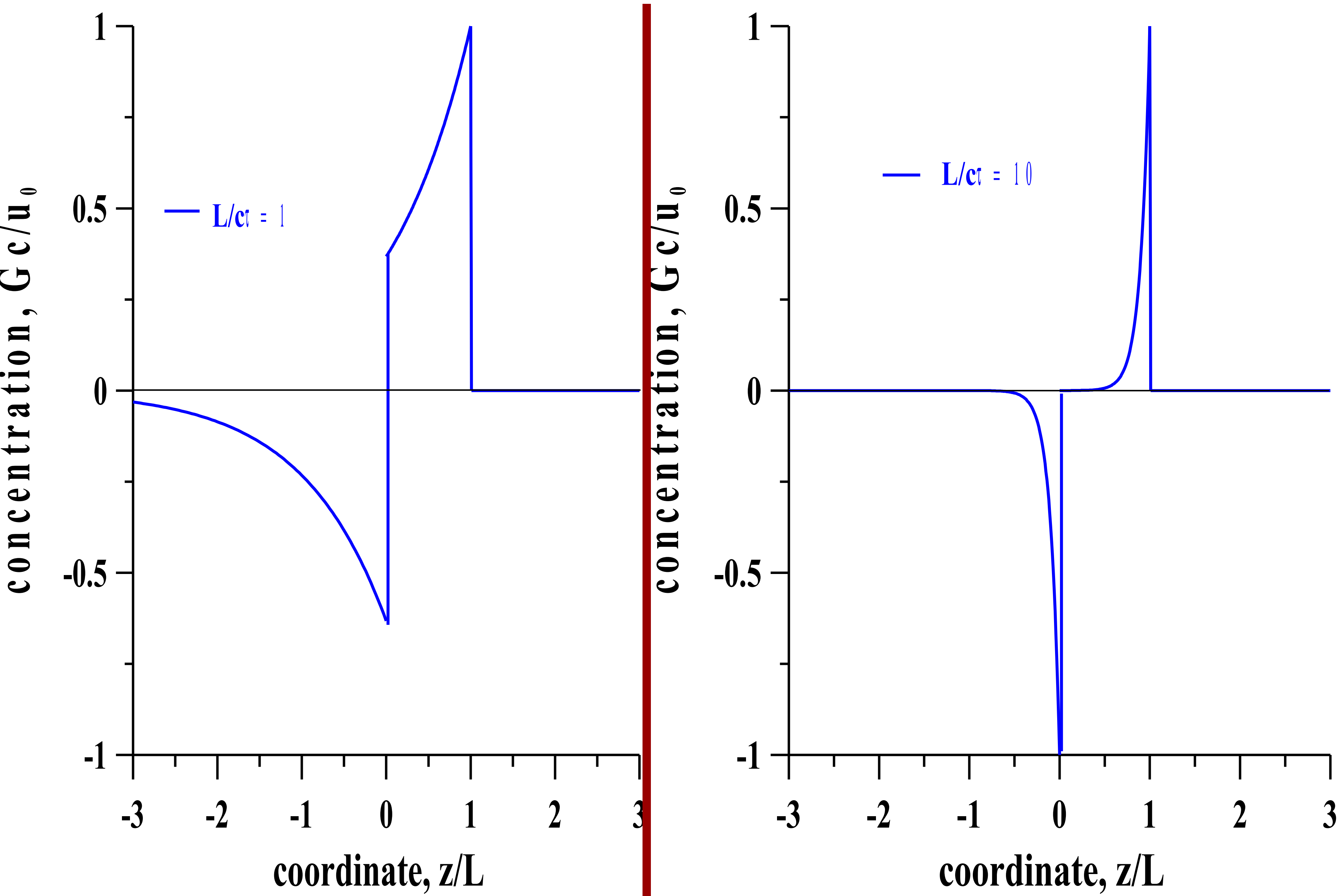


D = 100 m^2/s, τ = ∞ s, u_0 = 21 cm/s
diffusion
x, km
t, hours
0
1
2
3
4
5
0.9
1
1.1
1.2
1.3
1.4
Concentration, Γ / Γ_0
k(x-ct)
0.8
0.9
1
1.1
1.2
1.3
0
5
10
15
20
$t=\pi/(3\omega)$
$t=2\pi/(3\omega)$
$t=4\pi/(3\omega)$
$t=2\pi/\omega$

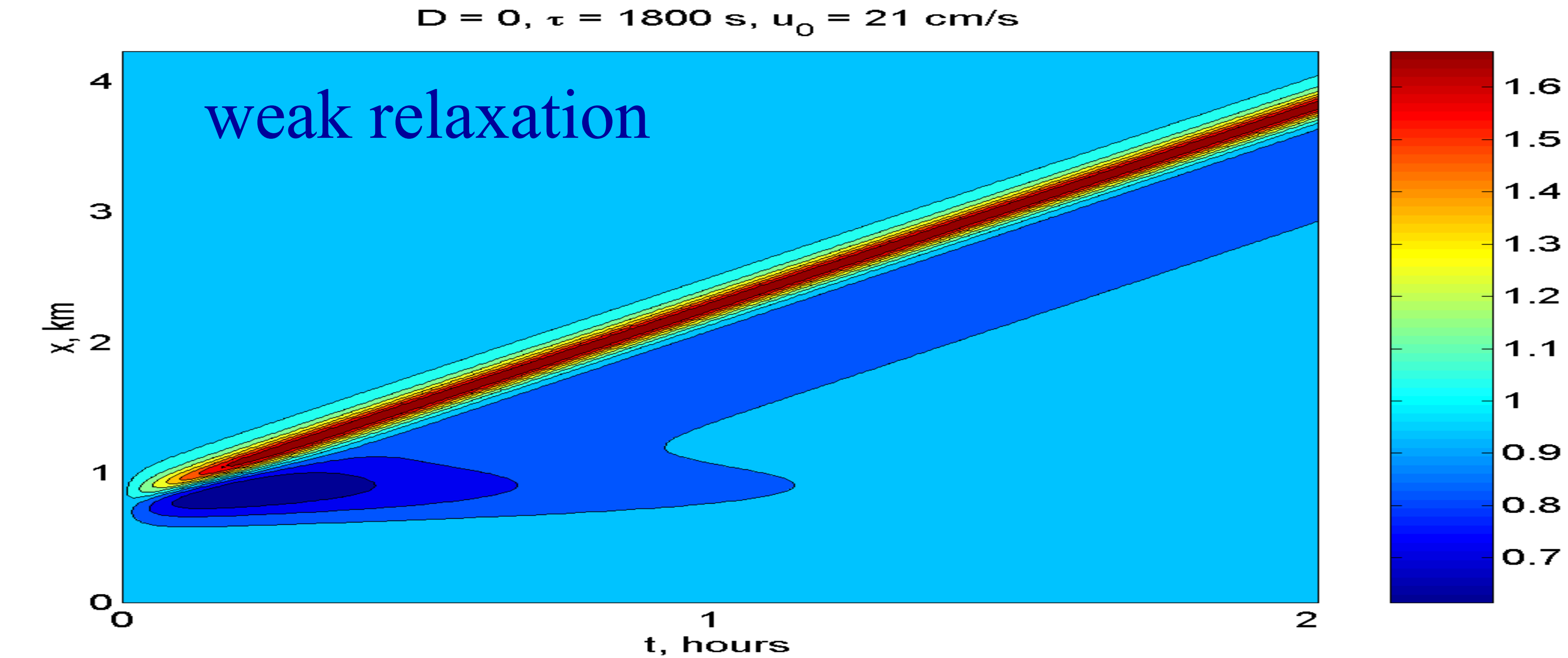


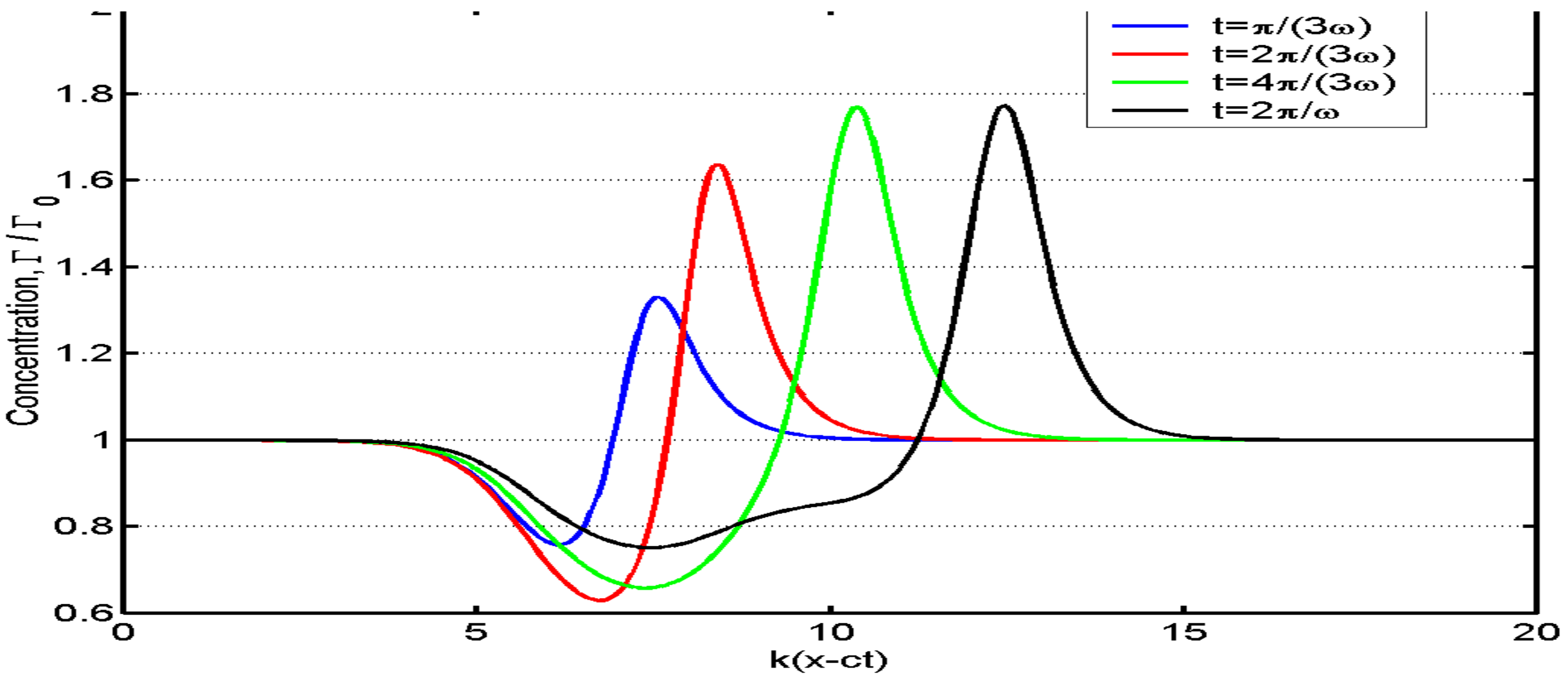
Relaxation
Back phase
L/cτ = 0.1
concentration, Gc/u$_0$
coordinate, z/L
1
0.5
0
-0.5
-3
-2
-1
0
1
2
3
$G(z)=\begin{cases} -\frac{u_0}{c}\exp(z/c\tau)(1-\exp(-L/c\tau)), \\ \frac{u_0}{c}\exp\left(\frac{z-L}{c\tau}\right) \\ 0, \end{cases}$

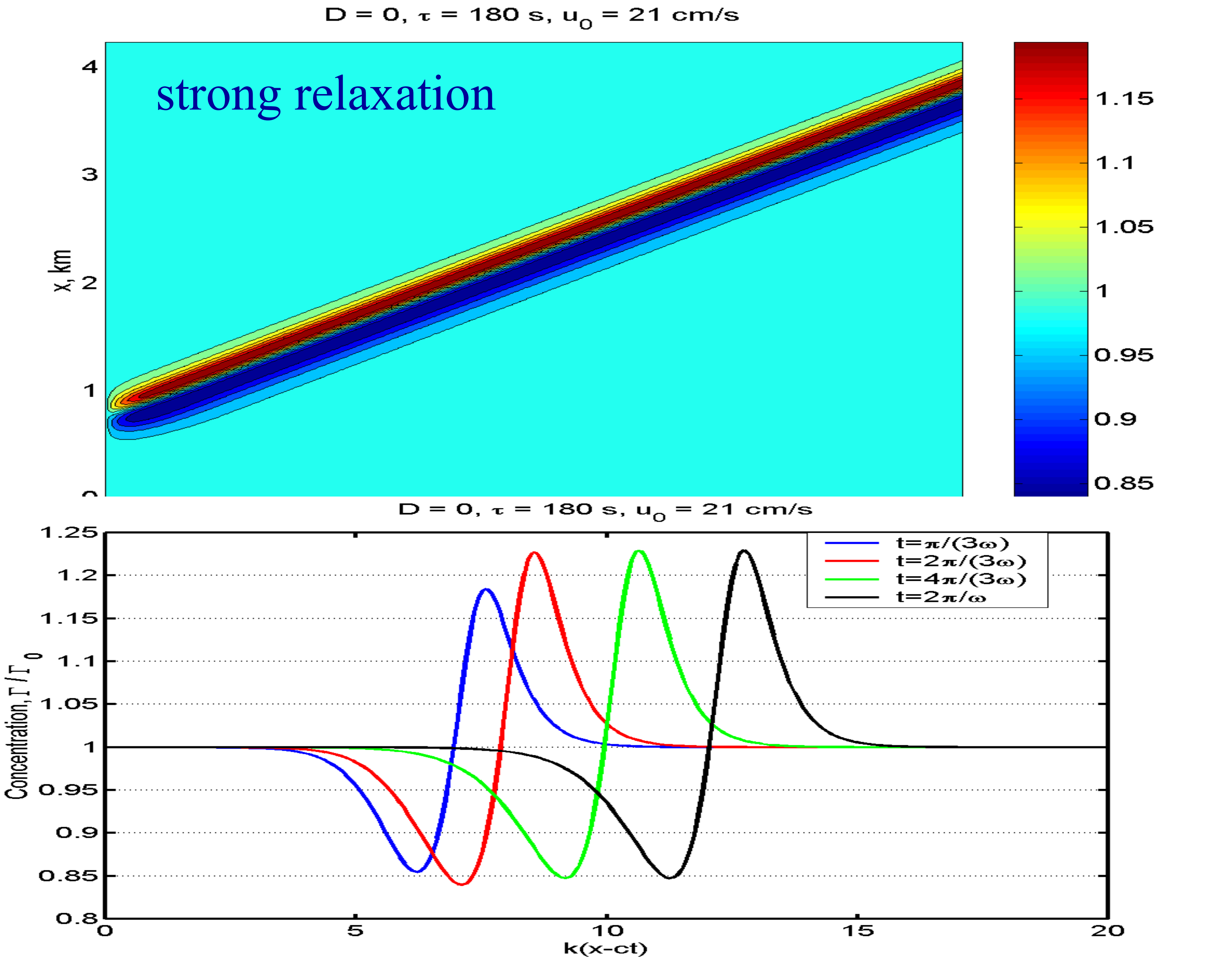
Relaxation



concentration, G c/u0
L/cτ = 1
coordinate, z/L
concentration, G c/u0
L/cτ = 10
coordinate, z/L


D = 0, τ = 1800 s, u0 = 21 cm/s
weak relaxation
x, km
t, hours
0
1
2
3
4
0
1
2
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7


t=π/(3ω)
t=2π/(3ω)
t=4π/(3ω)
t=2π/ω
Concentration, Γ / Γ0
k(x-ct)
0.6
0.8
1
1.2
1.4
1.6
1.8
0
5
10
15
20



D = 0, τ = 180 s, u0 = 21 cm/s
strong relaxation
x, km
0
1
2
3
4
1.15
1.1
1.05
1
0.95
0.9
0.85
D = 0, τ = 180 s, u0 = 21 cm/s
t=π/(3ω)
t=2π/(3ω)
t=4π/(3ω)
t=2π/ω
Concentration, Γ / Γ0
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0
5
10
15
20
k(x-ct)

Surfactant "wake" of wave packet

$$u(x,t) = u_0 A(x - c_{gr}t)\exp\left(ik(x - c_{ph}t)\right) + c.c.$$

Two terms with short and long wavelengths:

$$\Lambda = \lambda \frac{c_{gr}}{c_{ph} - c_{gr}}$$

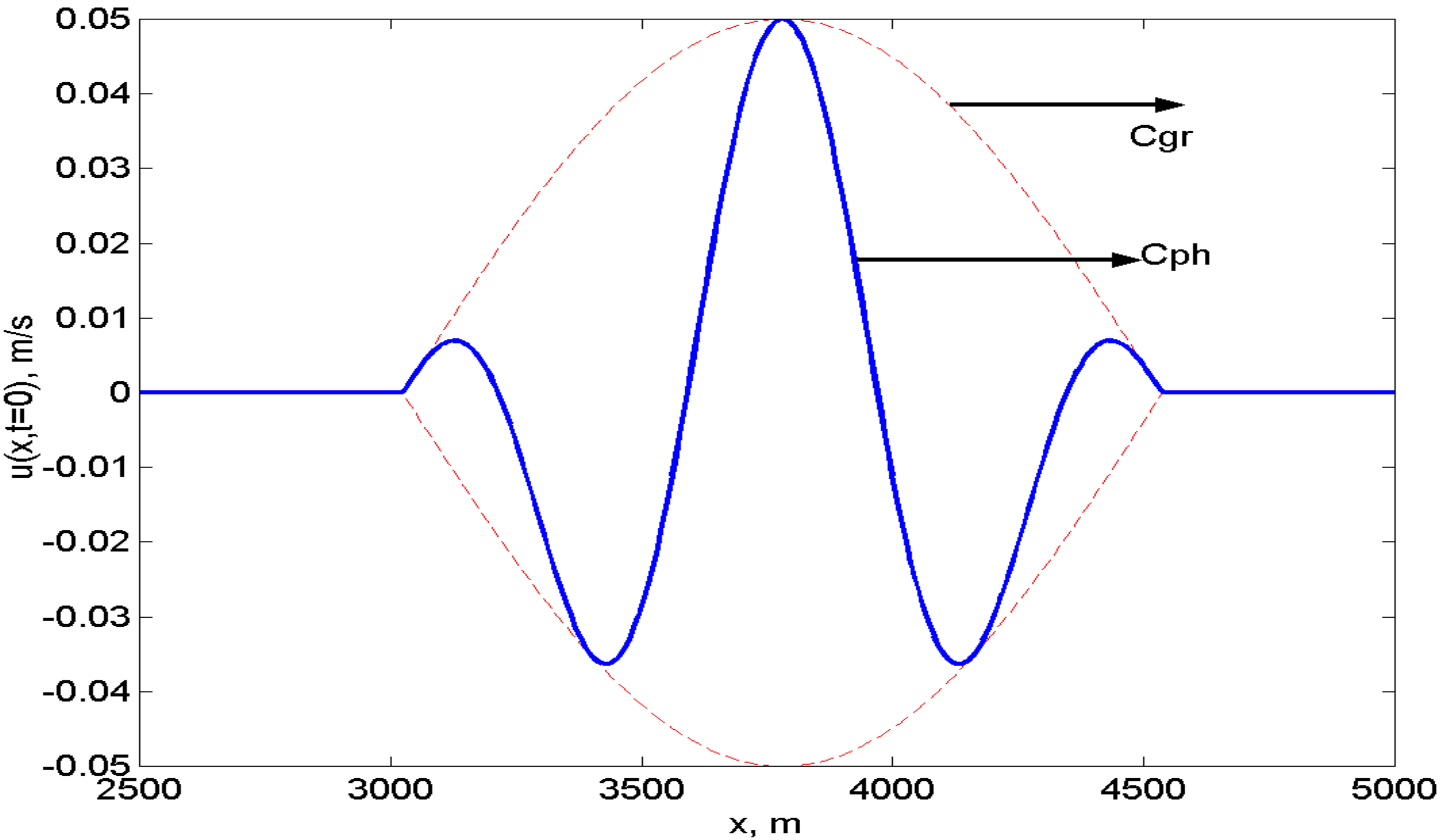
$$G(x,t) = \frac{u_0}{c_{gr}}\left\{A(x - c_{gr}t)\exp\left(ik(x - c_{ph}t)\right) - A(x)\exp(ikx) + \right.$$

$$\left. + ik\left(1 - \frac{c_{ph}}{c_{gr}}\right)\exp\left(ikx\left[1 - \frac{c_{ph}}{c_{gr}}\right]\right)\int_x^{x - c_{gr}t} A(x')\exp\left(\frac{ikc_{ph}x'}{c_{gr}}\right)dx'\right\} + c.c$$

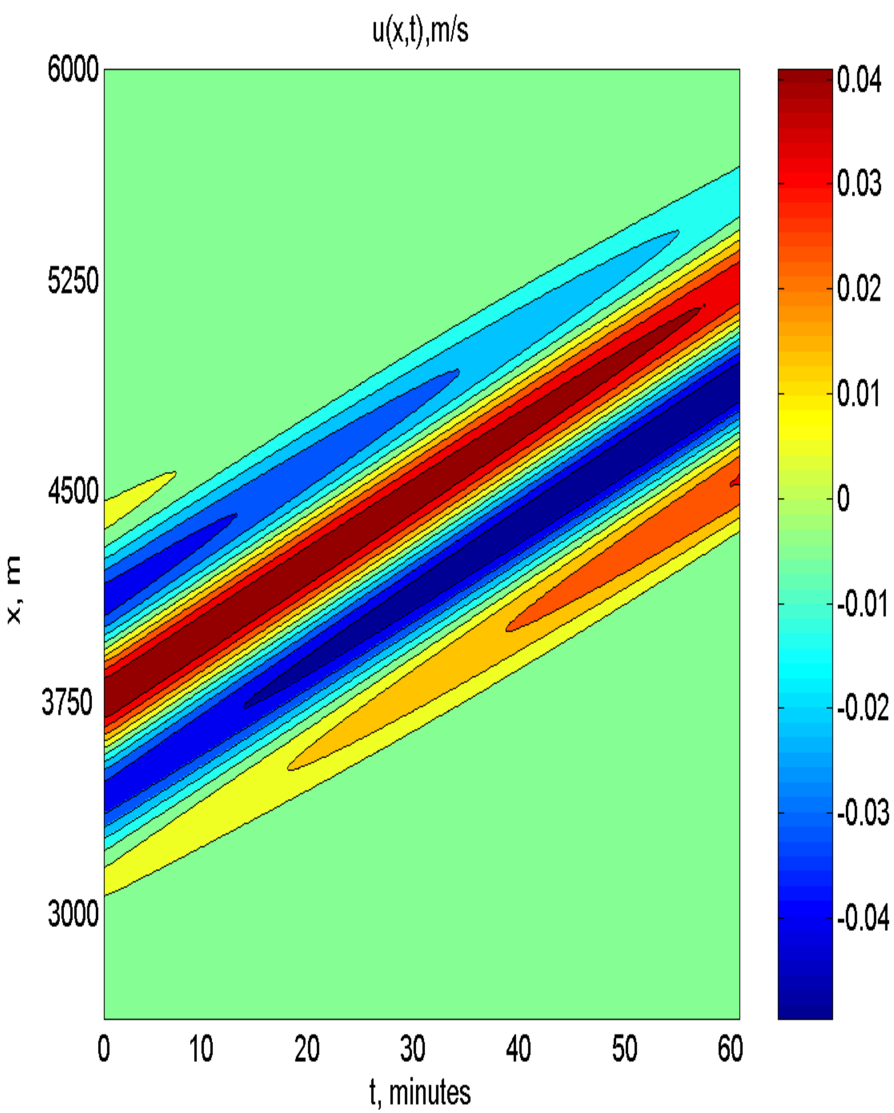
Internal Wave Packet

T = 30 min

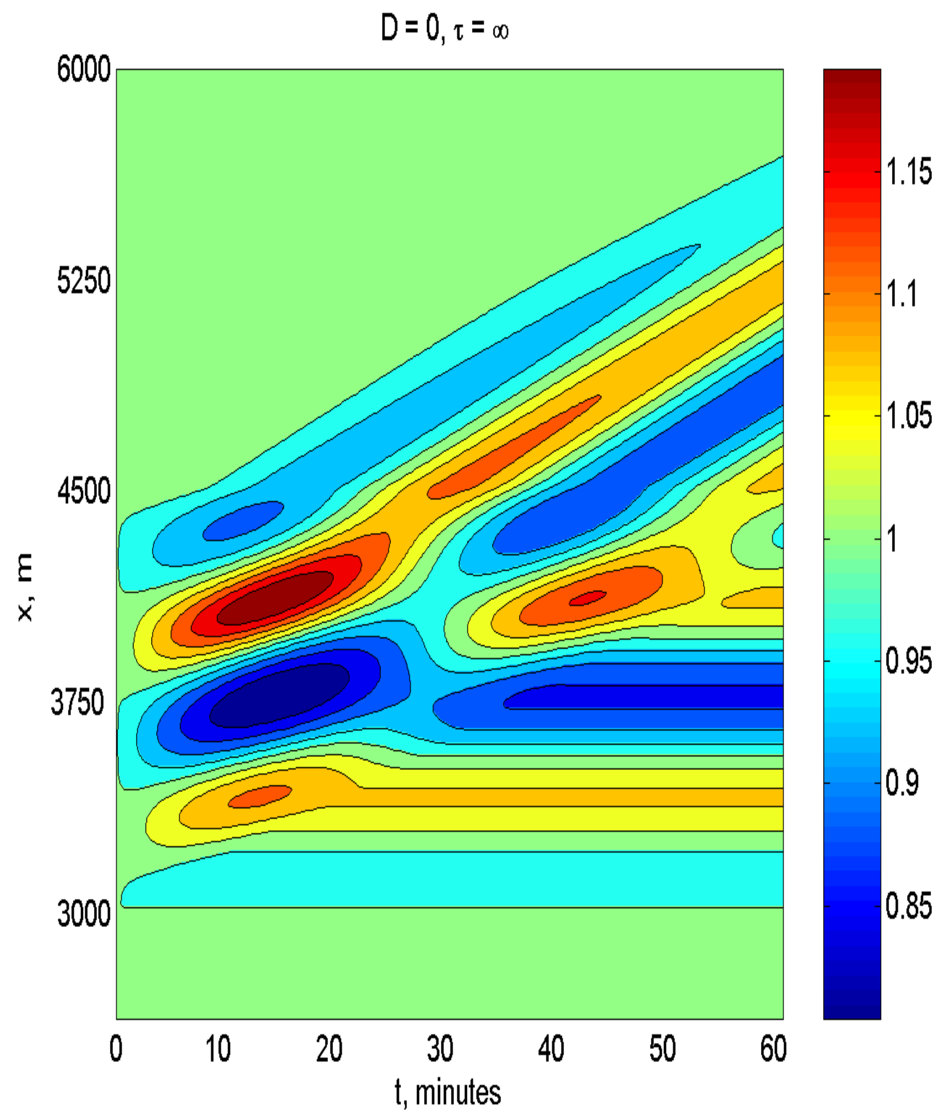
$c_{ph} = 0.42$ m/s $\quad c_{gr} = 0.33$ m/s

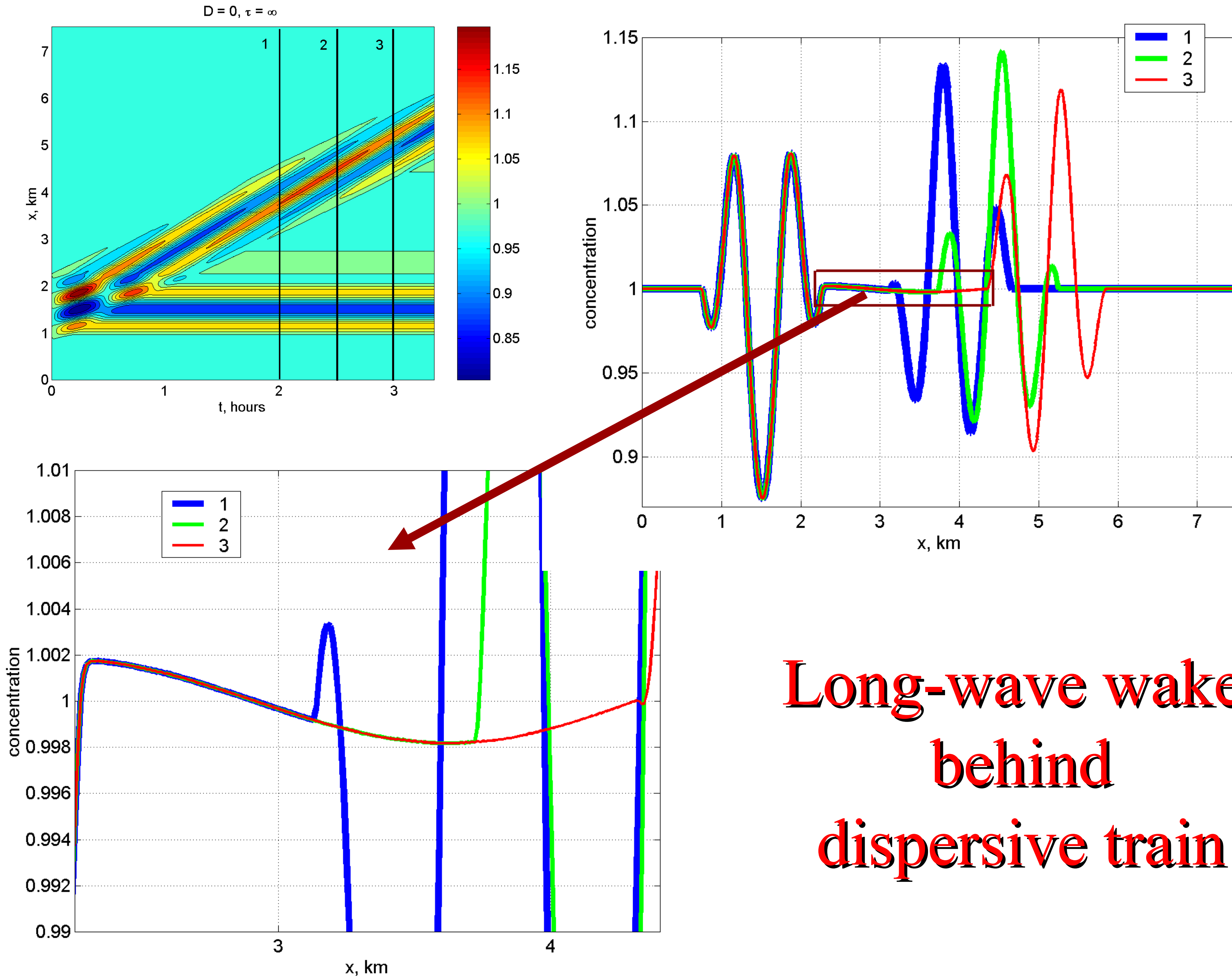


Surface Current



Surfactant Concentration



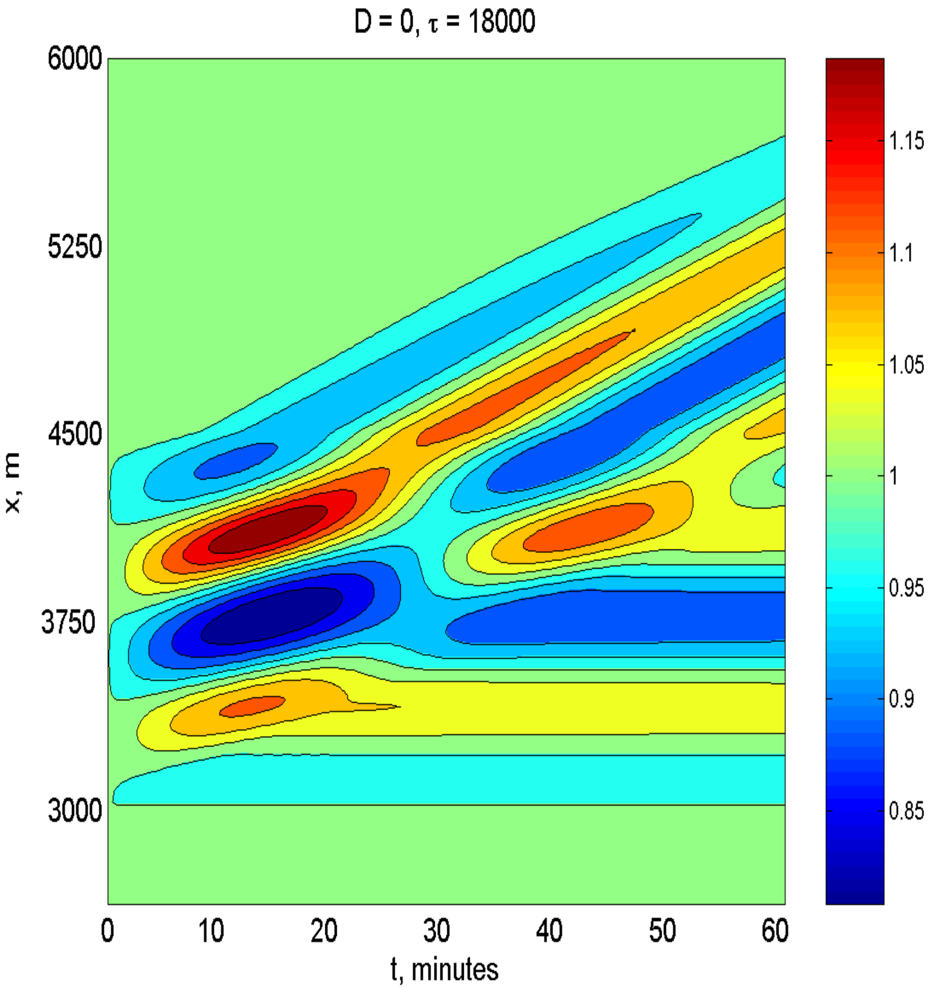


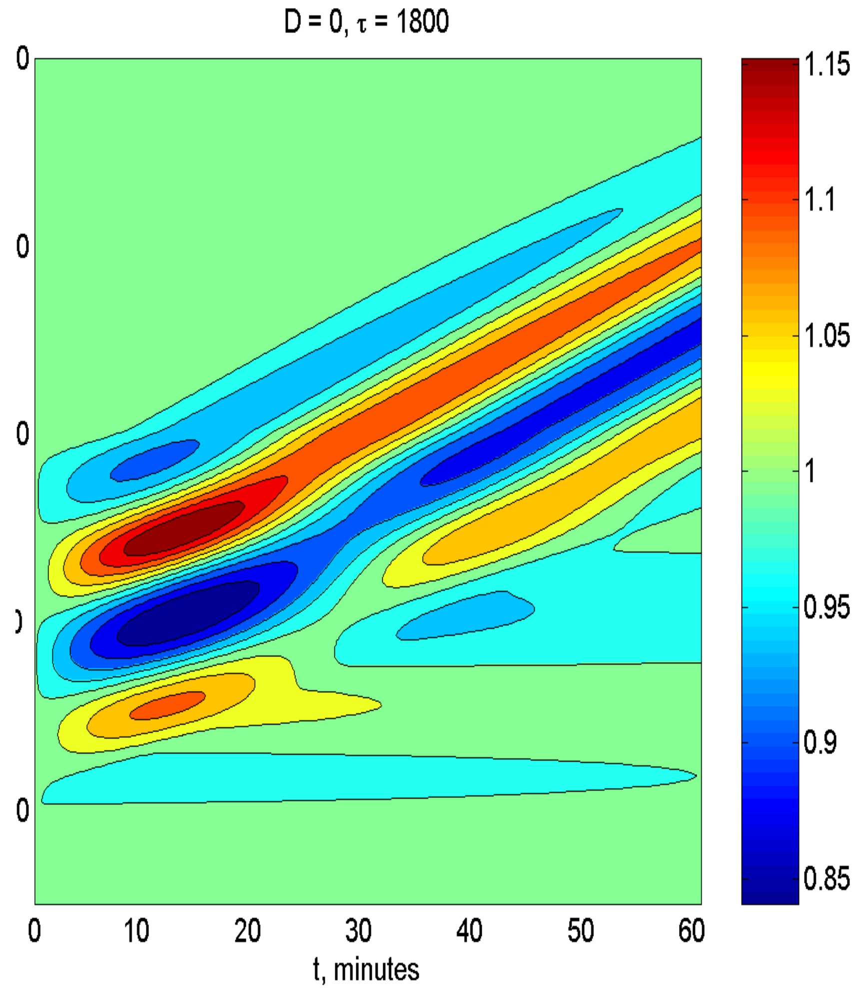
Long-wave wake behind dispersive train

Relaxation

weak

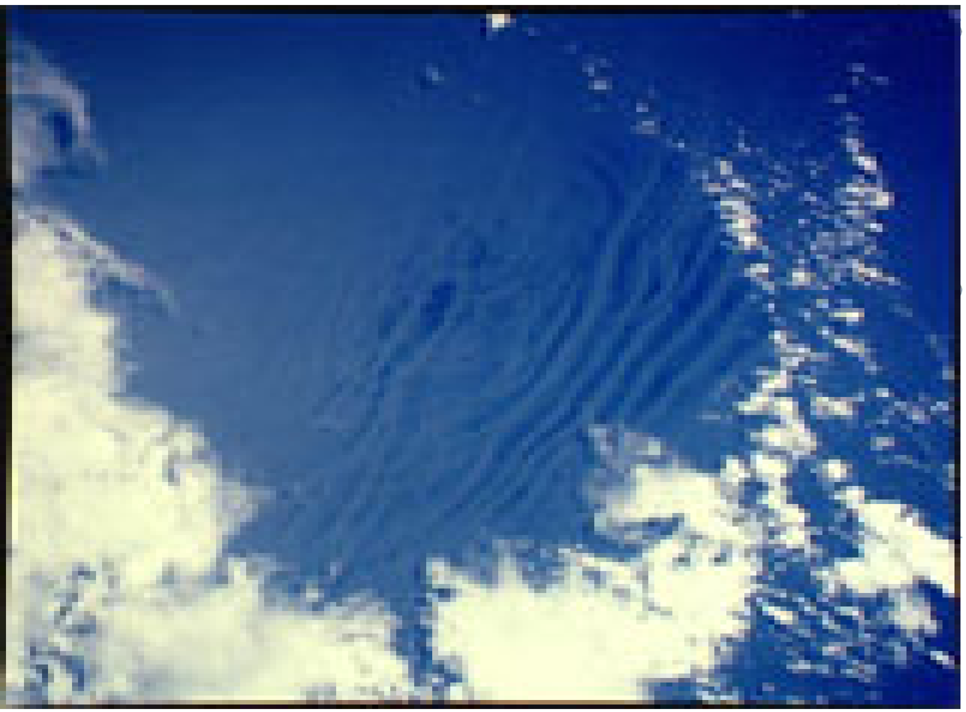
strong

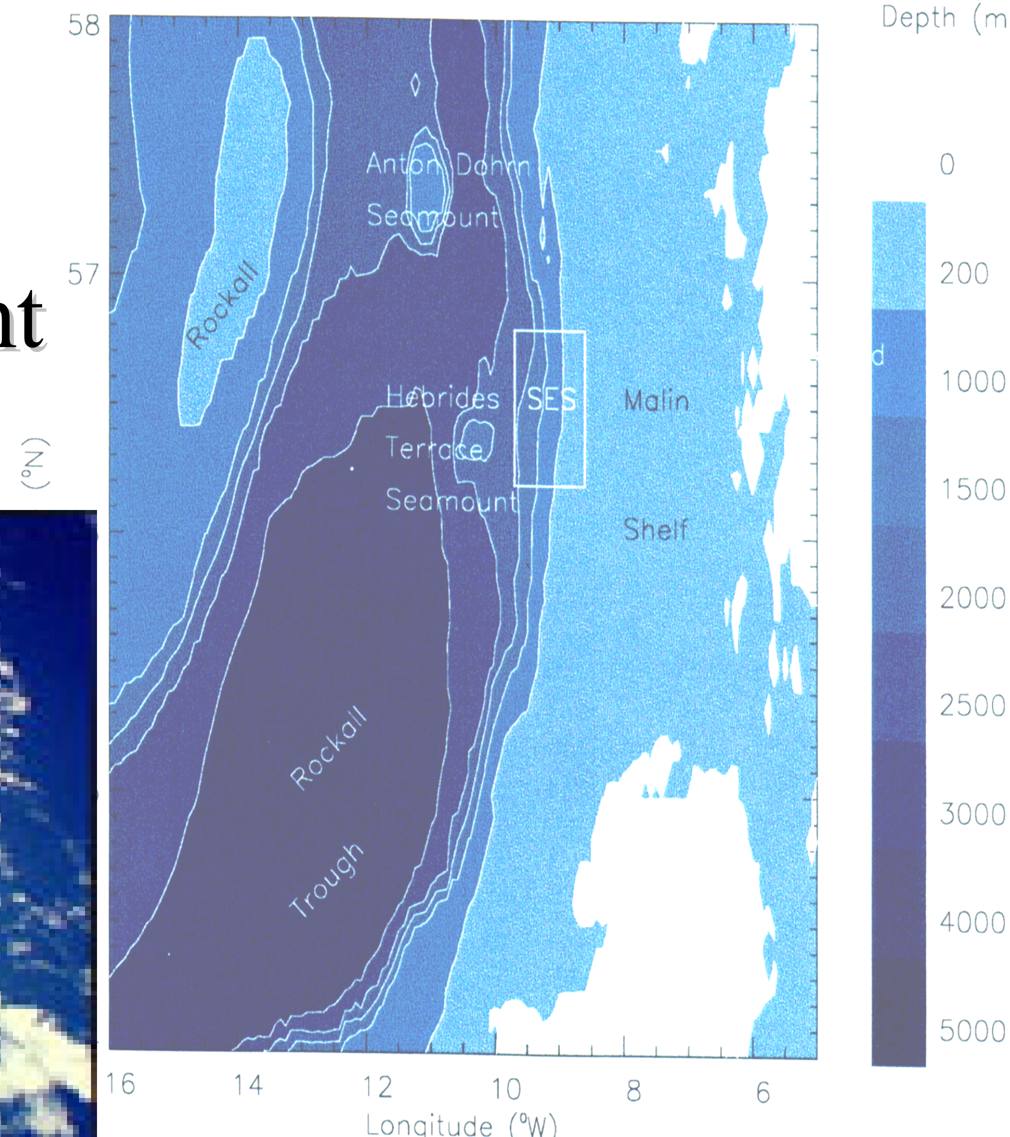




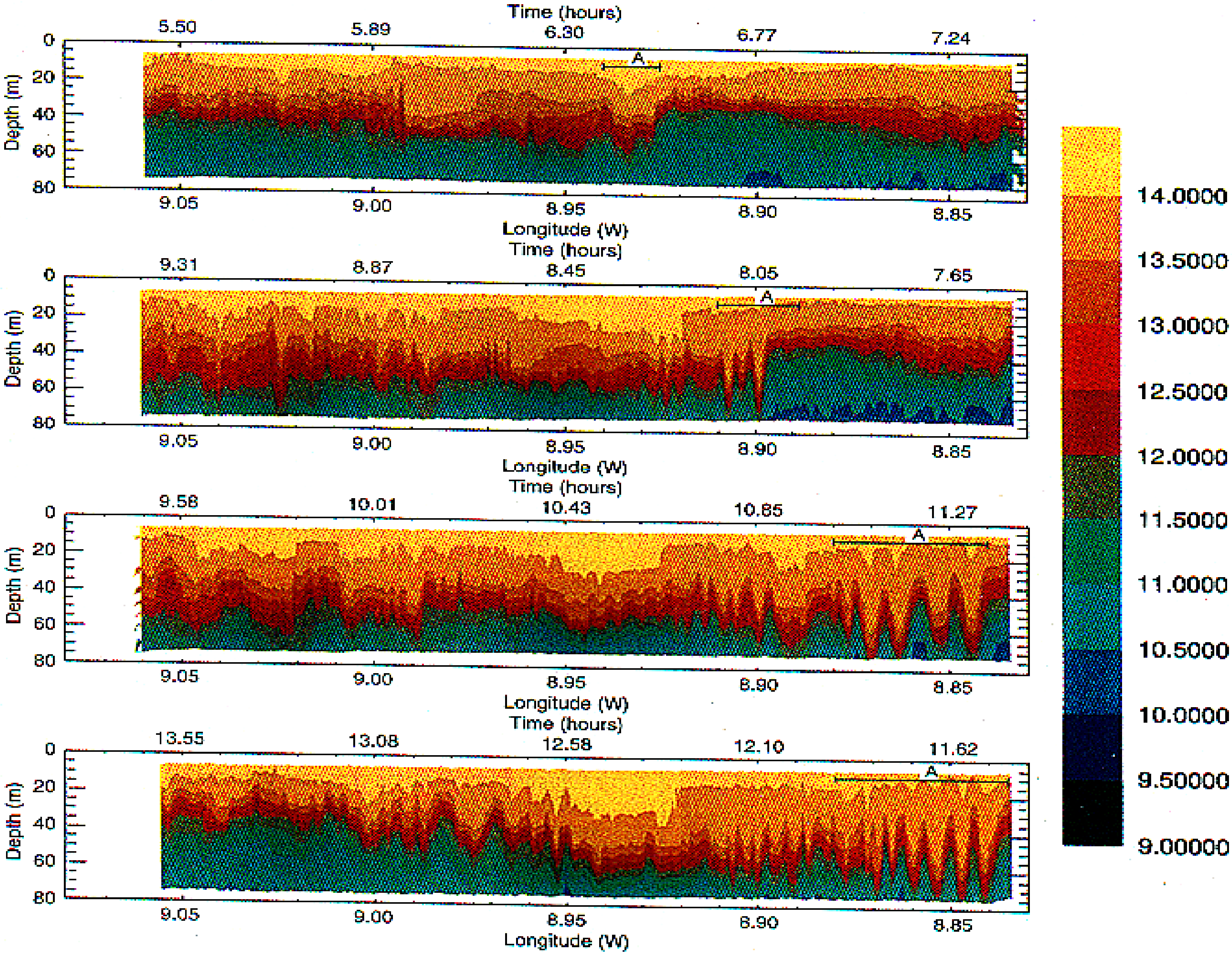
SESAME-1996

Shelf Edge Study
Acoustic Measurement
Experiment)



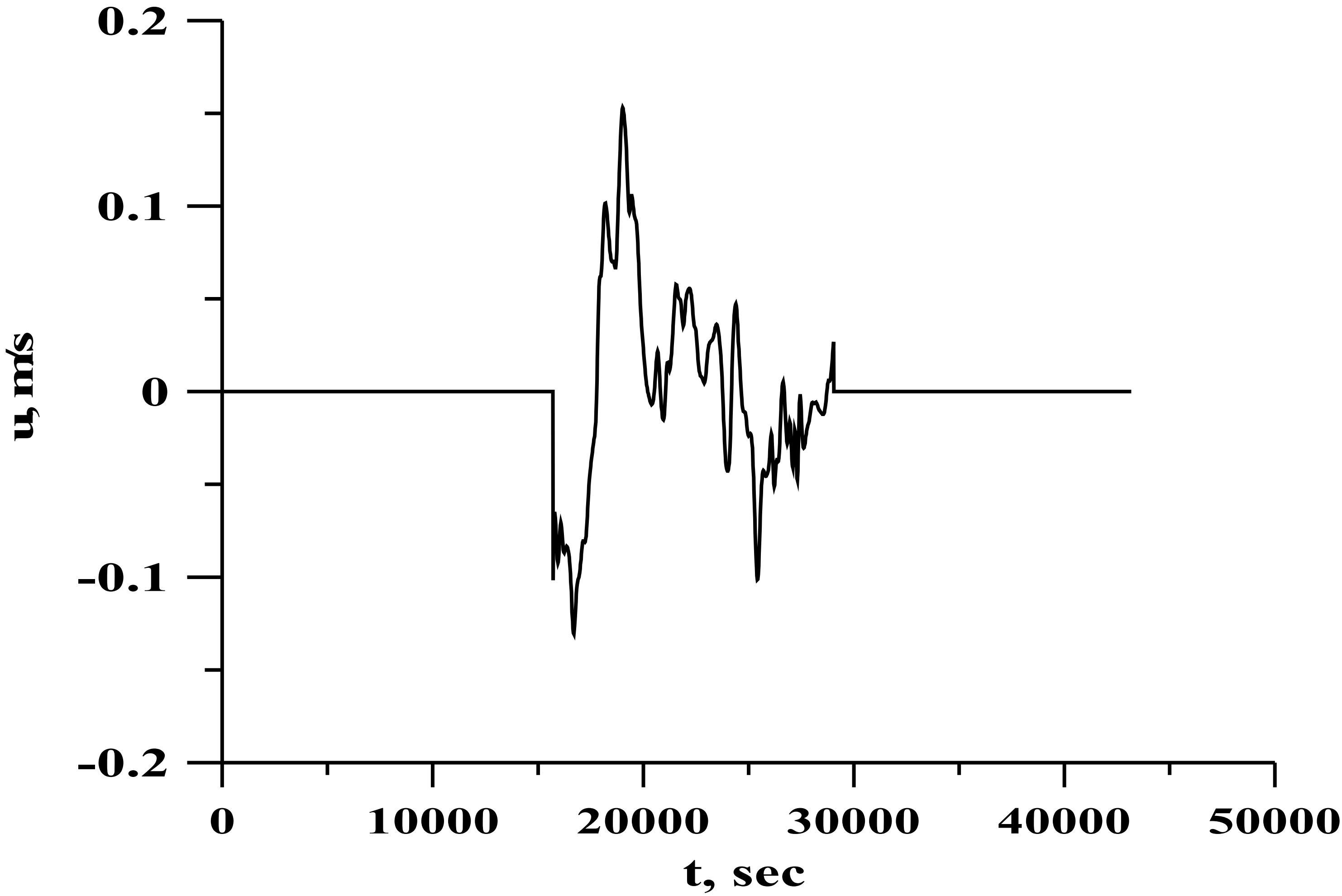


Ship tows: Thermistor chain and ADCP (DERA, NRL)

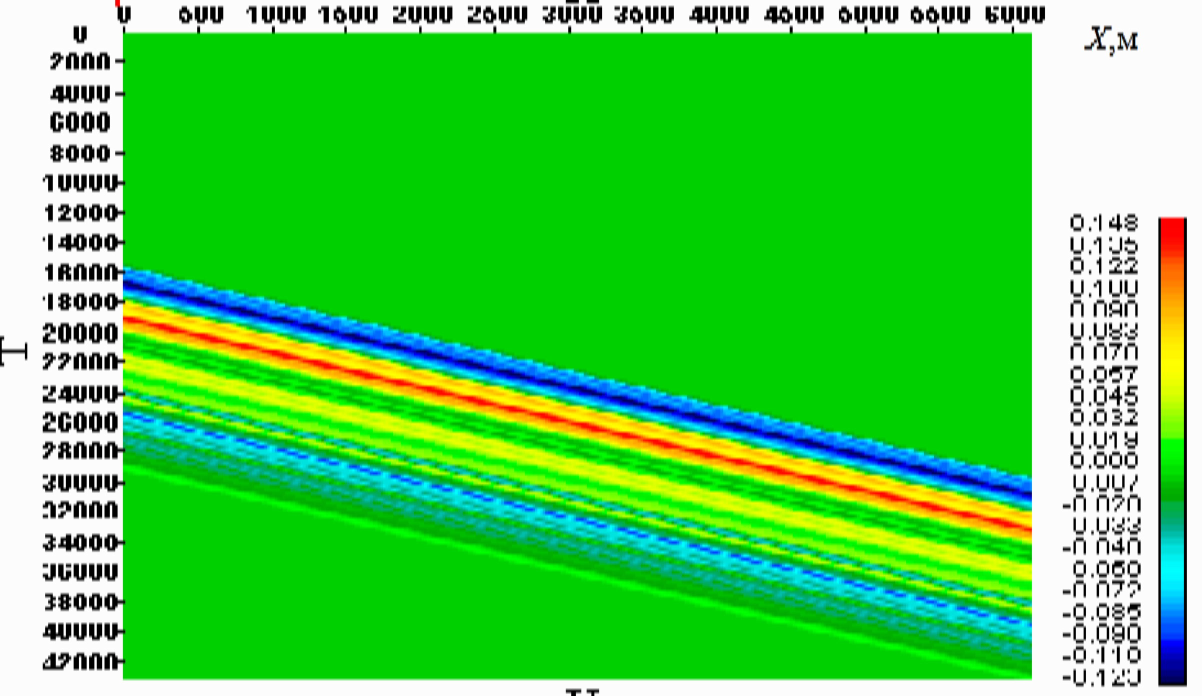


Internal Waves (Malin Shelf, 1996)

SESAME, Malin Shelf

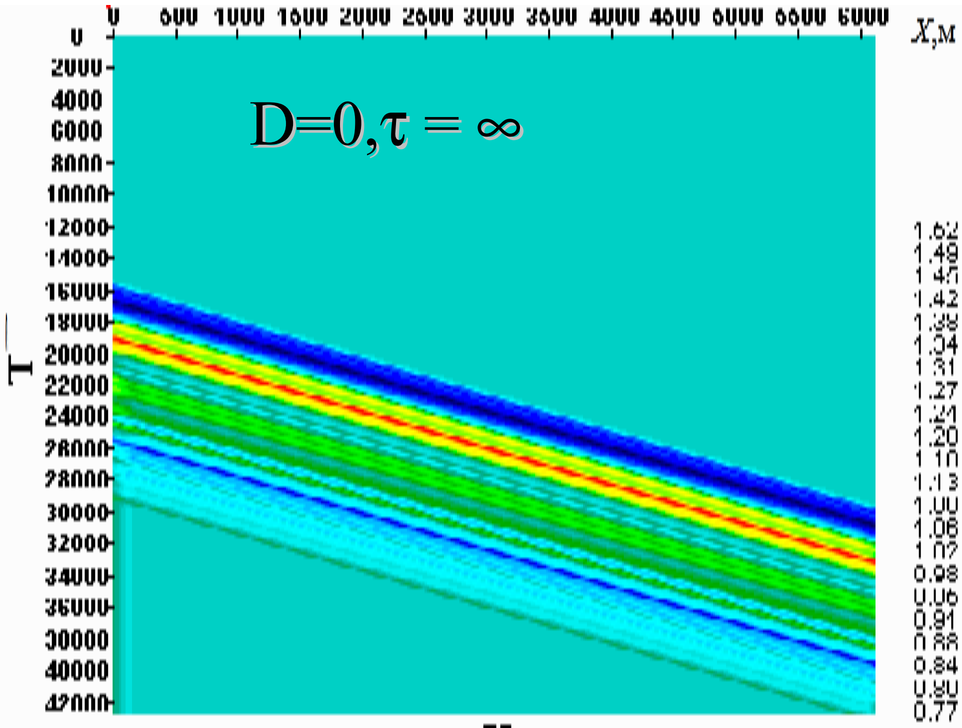


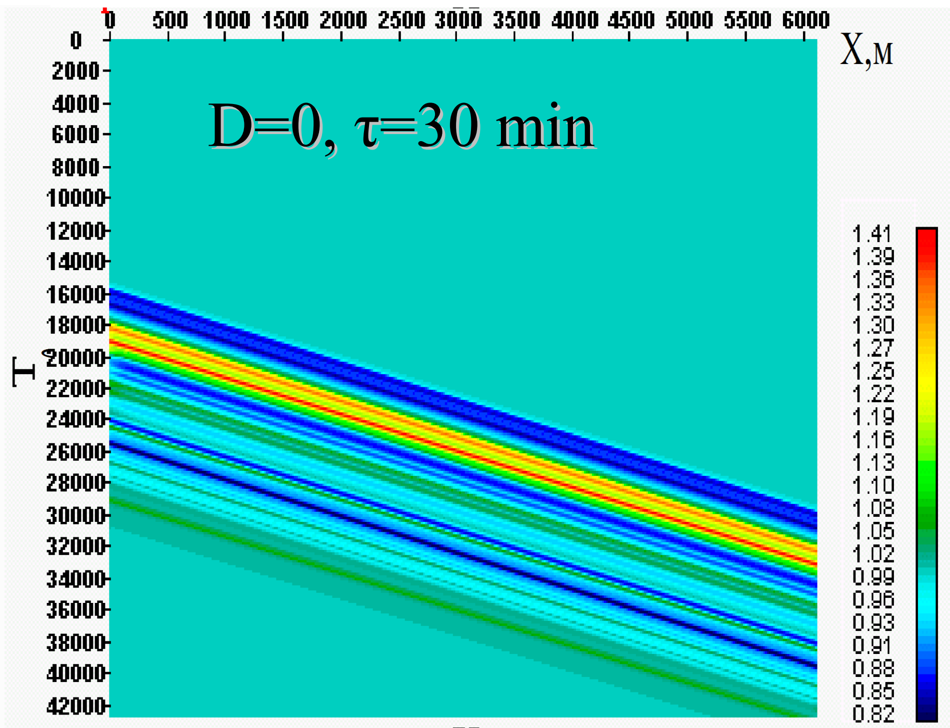
Computed surface velocity



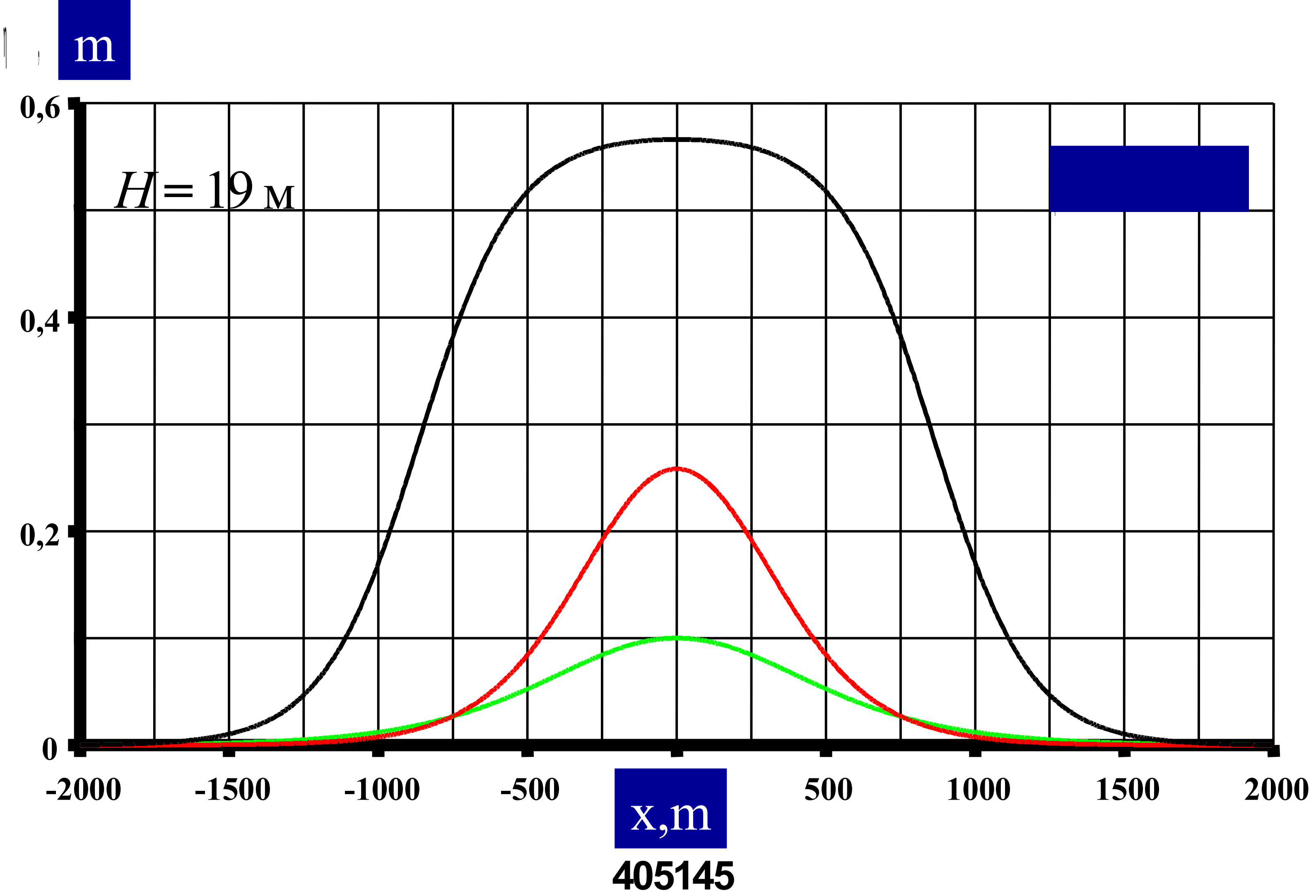
Surface Velocity

Concentration

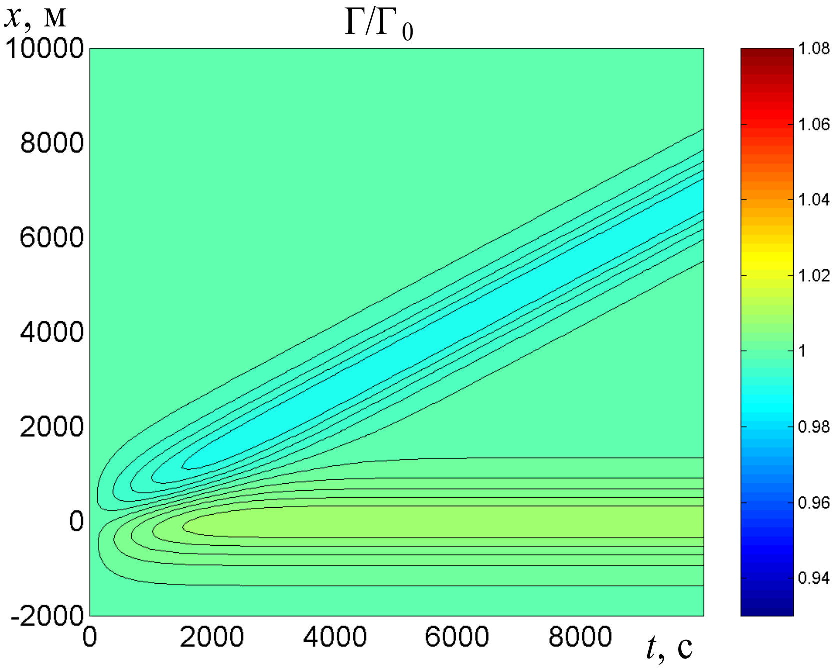


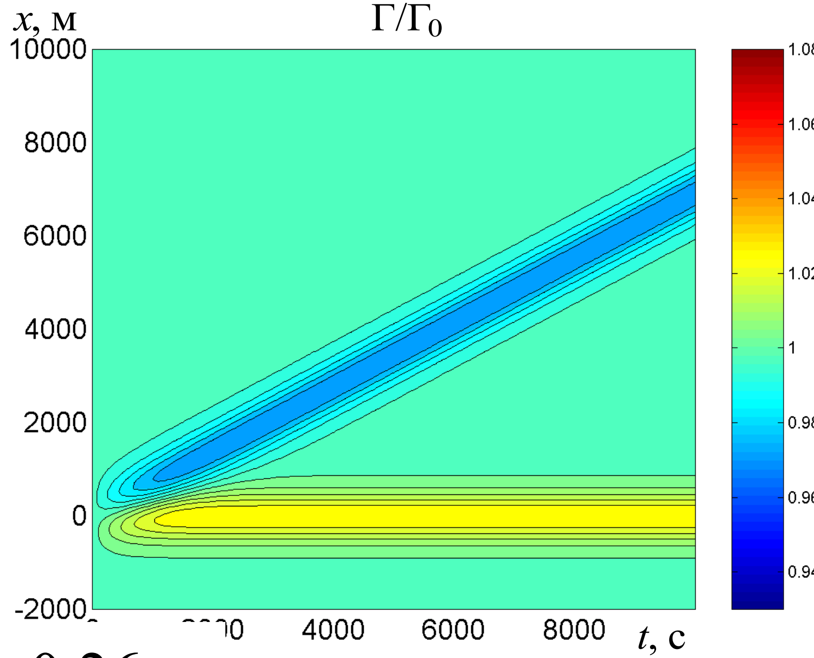



Russia
International boundary
Railroad
National capital
Road
0 250 500 750 Kilometers
0 250 500 750 Miles
Lambert Conformal Conic Projection, SP 47/62 N
Boundary representation is not necessarily authoritative.
ARCTIC OCEAN
NORTH ATLANTIC OCEAN
Norwegian Sea
Barents Sea
Kara Sea
Laptev Sea
East Siberian Sea
Chukchi Sea
Bering Sea
NORTH PACIFIC OCEAN
Sea of Okhotsk
Sea of Japan
Yellow Sea
East China Sea
North Sea
Baltic Sea
Gulf of Bothnia
Black Sea
Caspian Sea
Aral Sea
Lake Balkhash
Lake Baikal
Persian Gulf
SVALBARD (NORWAY)
FRANZ JOSEF LAND
NOVAYA ZEMLYA
SEVERNAYA ZEMLYA
NEW SIBERIAN ISLANDS
Wrangel Island
Sakhalin Island
KURIL ISLANDS
Occupied by Soviet Union in 1945, administered by Russia, claimed by Japan.
Arctic Circle
U.S.
UNITED KINGDOM
NORWAY
SWEDEN
FINLAND
GERMANY
POLAND
BELARUS
UKRAINE
ROMANIA
TURKEY
SYRIA
IRAQ
IRAN
SAUDI ARABIA
KAZAKHSTAN
UZBEKISTAN
TURKMENISTAN
KYRGYZSTAN
AFGHANISTAN
CHINA
MONGOLIA
JAPAN
NORTH KOREA
SOUTH KOREA
Amsterdam
Oslo
Copenhagen
Stockholm
Helsinki
Tallinn
Riga
Vilnius
Minsk
Warsaw
Berlin
Kiev
Bucharest
Chisinau
Ankara
Tbilisi
Yerevan
Baku
Tehran
Baghdad
Ashgabat
Tashkent
Bishkek
Almaty
Dushanbe
Moscow
St. Petersburg
Murmansk
Arkhangel'sk
Vorkuta
Salekhard
Noril'sk
Igarka
Dikson
Tiksi
Yakutsk
Magadan
Pevek
Anadyr'
Provideniya
Cherskiy
Petropavlovsk-Kamchatskiy
Okhotsk
Khabarovsk
Vladivostok
Yuzhno-Sakhalinsk
Blagoveshchensk
Chita
Irkutsk
Ulan-Ude
Bratsk
Krasnoyarsk
Novosibirsk
Omsk
Tomsk
Barnaul
Novokuznetsk
Surgut
Yekaterinburg
Chelyabinsk
Perm'
Kazan
Samara
Saratov
Volgograd
Rostov
Astrakhan
Orenburg
Ufa
Tyumen'
Tobol'sk
Harbin
Shenyang
Dalian
Qingdao
Seoul
P'yongyang
Pusan
Tokyo
Fukuoka
Aomori
Lena
Yenisey
Ob'
Amur



Internal Soliton

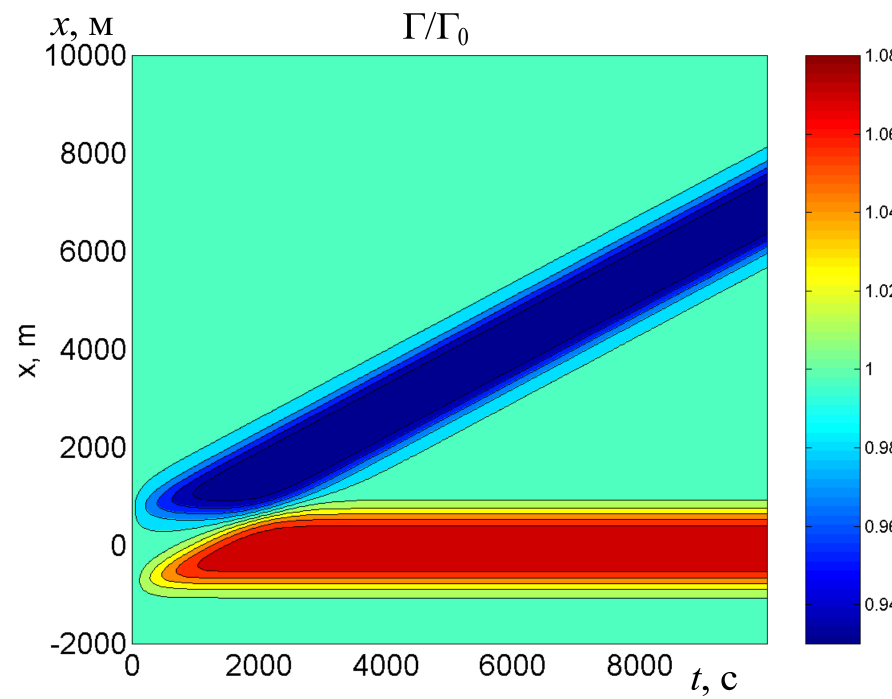




A = 0,26 m

A = 0,11 m

Concentration



A = 0,57 m

Conclusions:

- Analytical Tests
- Unsteady effects
- Relaxation and Diffusion
- Influence on slick formation

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Pollutant Dynamics in Small Rivers

Rhum Production

Pollutant Dynamics in Small Rivers

1D

$$\frac{\partial C}{\partial t} + u(x,t)\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} - \frac{C}{\tau}$$

Pollutant Source at x = 0

$$C(x=0,t) = C_0(t)$$

or

$$D\frac{\partial C}{\partial x} = Q(t)$$

Far from Source $C(x \rightarrow \infty, t) \rightarrow 0$

Initial Condition (t = 0) $C(x,t=0)=0$

Analytical Methods for Simplified Situations
and
Numerical Modeling for Real Situations

The First Case

$$\frac{\partial C}{\partial t} + u(x,t)\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} - \frac{C}{\tau}$$

1. Constant Flow: U

2. Permanent Source: C_0

Solution: $$C(x) = C_0 \exp(-\mu x)$$

$$D\mu^2 + u\mu - \frac{1}{\tau} = 0$$

$$D\mu^2 + u\mu - \frac{1}{\tau} = 0$$

Parallel Flow
(u > 0)

i) No Diffusion

$$L = \mu^{-1} = u\tau$$

ii) Almost Insoluble

$$L \approx \sqrt{D\tau} \rightarrow \infty$$

Unsteady Process

$$D\mu^2 + u\mu - \frac{1}{\tau} = 0$$

Counter Flow
(u < 0)

i) No Diffusion

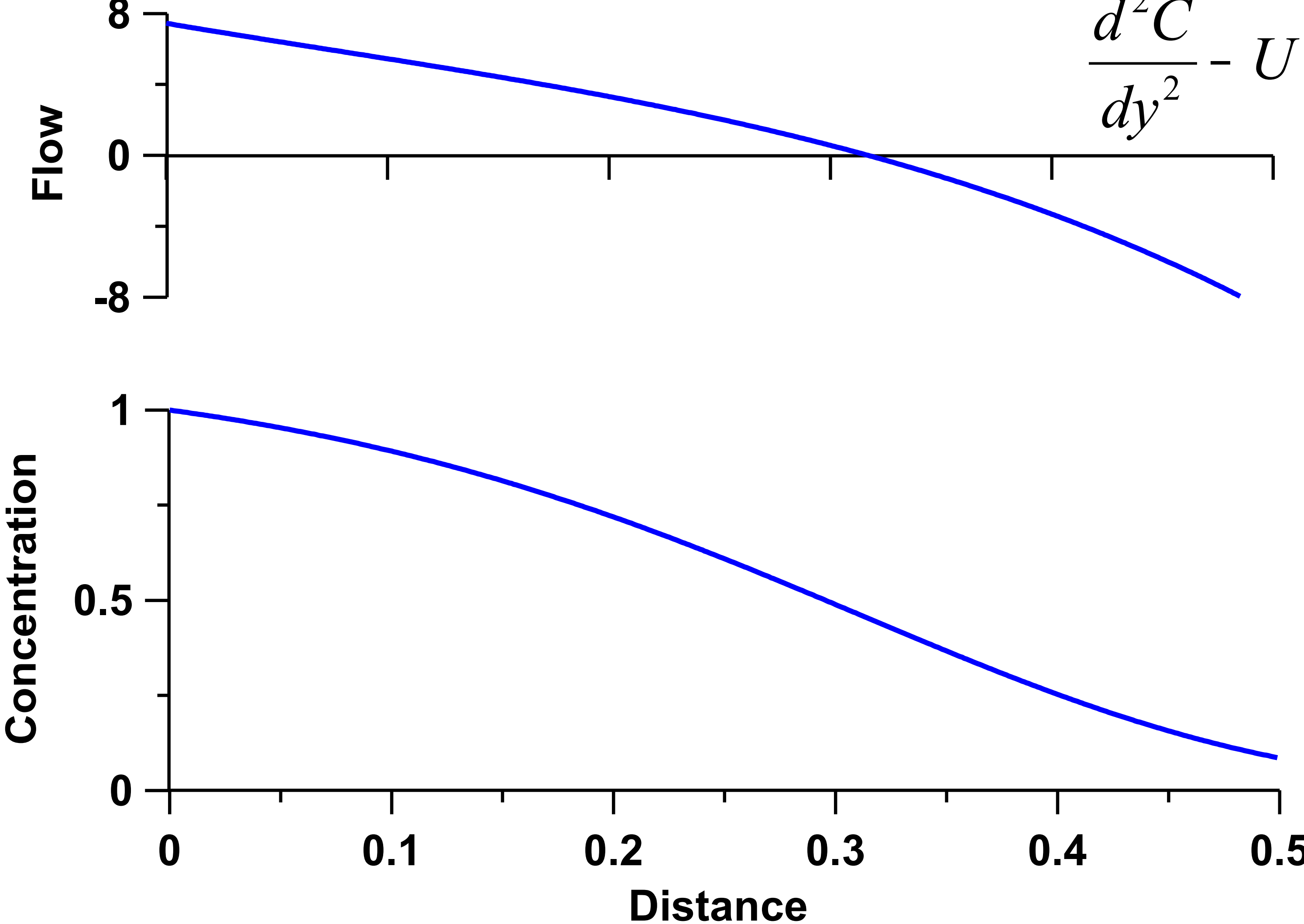
Unsteady Process

ii) Almost Insoluble

$$L = \frac{D}{|u|}$$

Steady Diffusion Relaxation Process in Variable Parallel Flow

$$\frac{d^2C}{dy^2} - U(y)\frac{dC}{dy} - C = 0$$



$$U = u\sqrt{\frac{\tau}{D}}$$

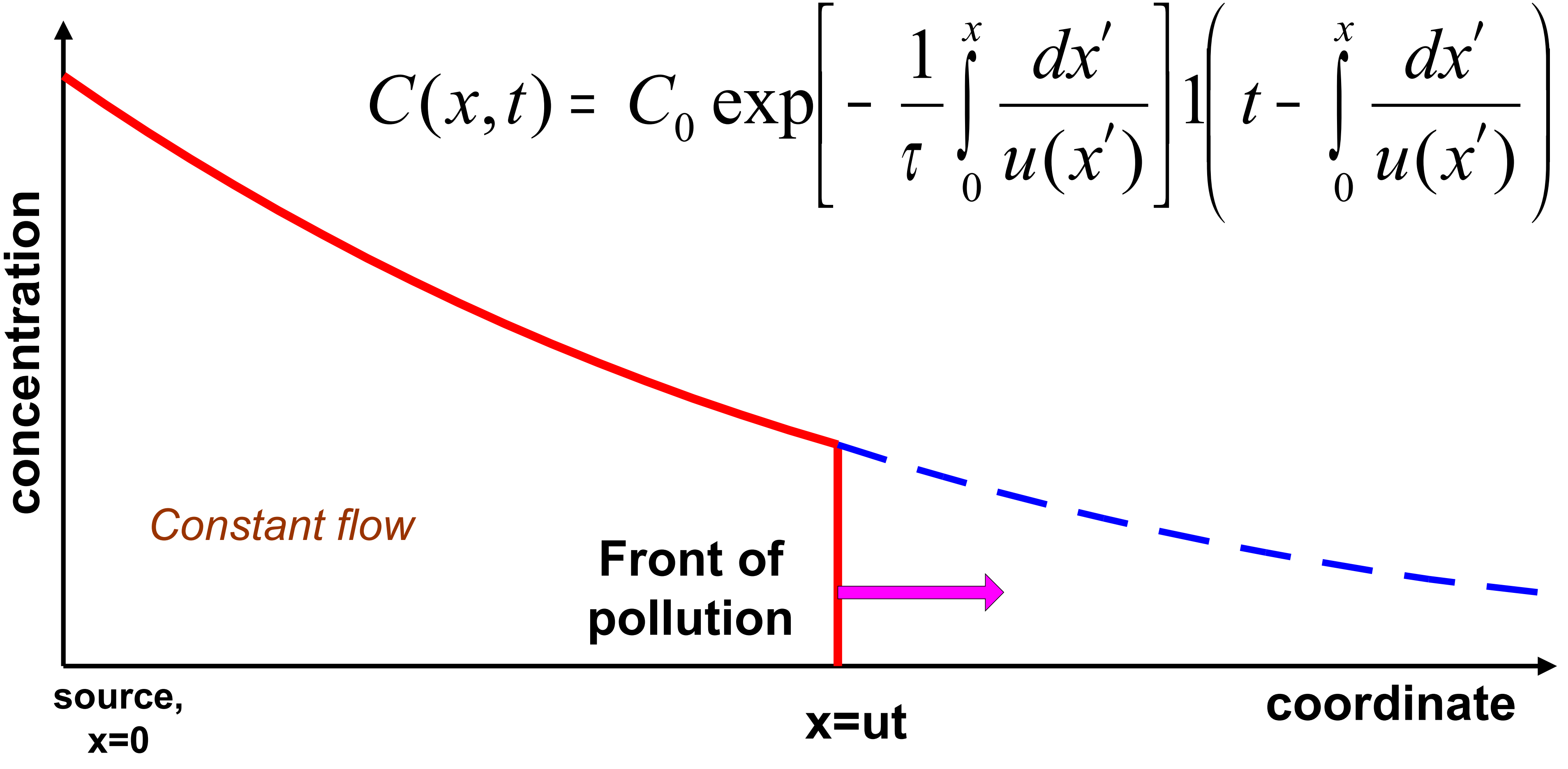
$$y = \frac{x}{\sqrt{D\tau}}$$

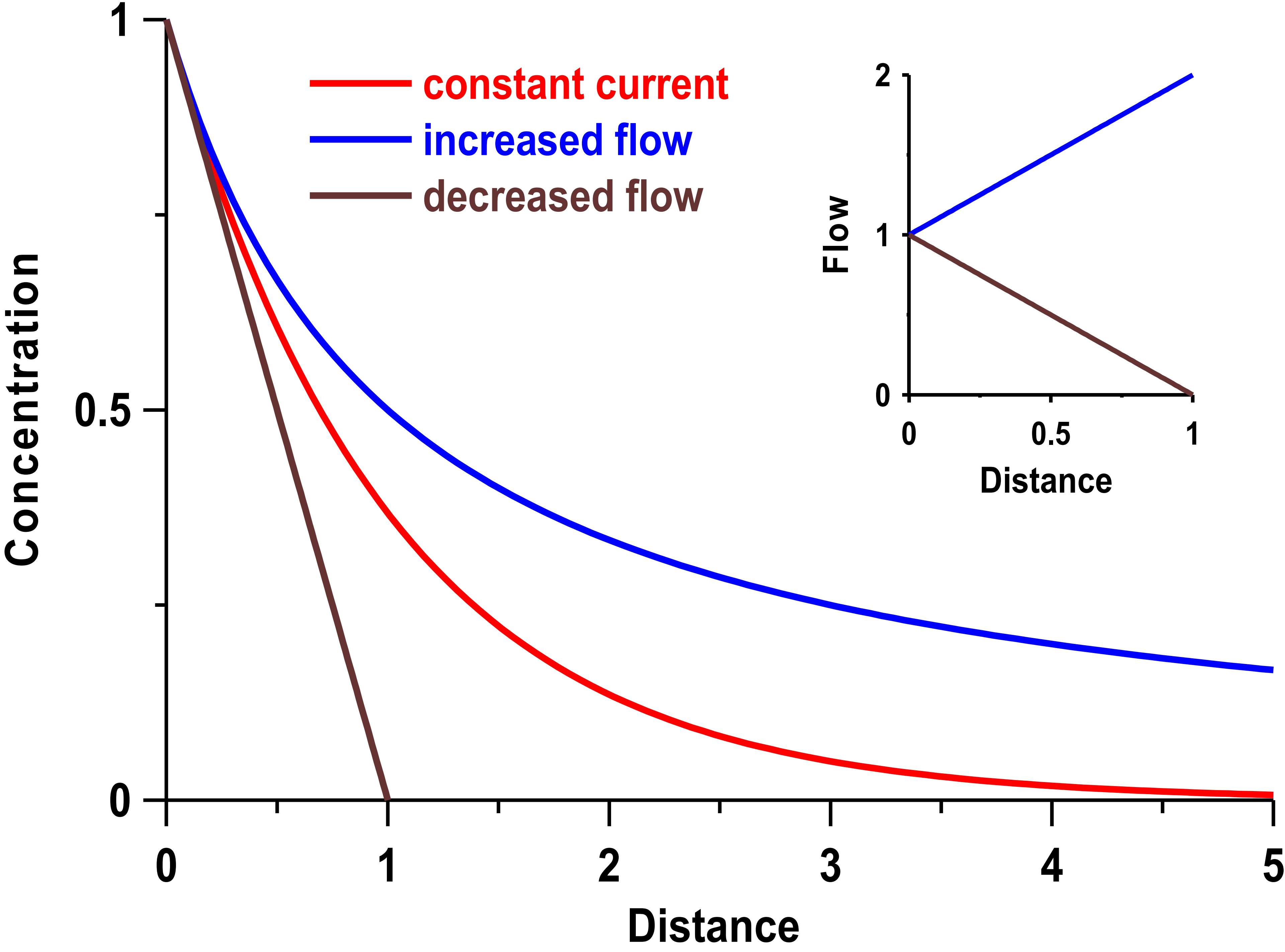
Unsteady Process in Variable Flow

Parallel flow, no diffusion

$$\frac{dx}{dt} = u(x,t)$$

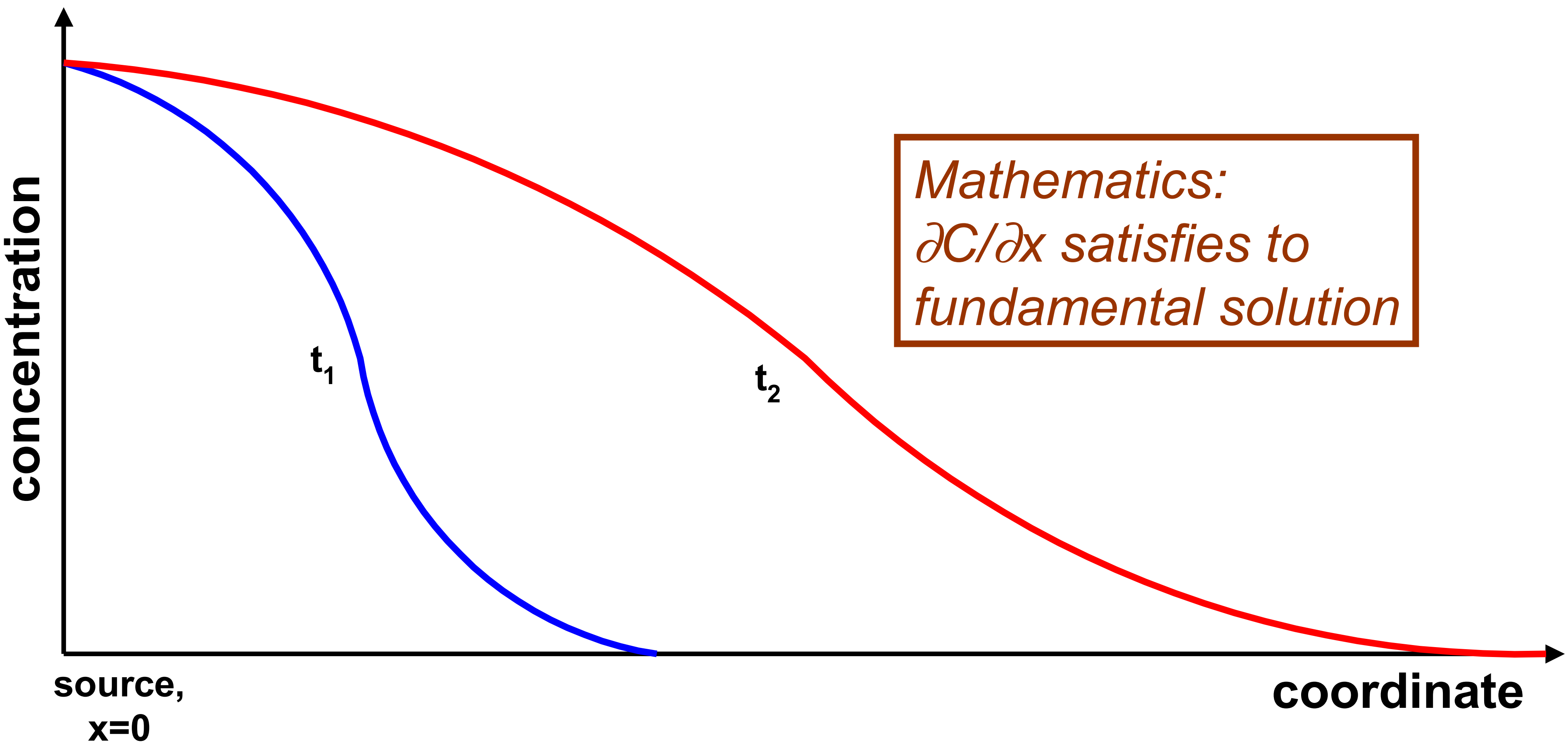
$$C(x,t) = C_0 \exp\left[- \frac{1}{\tau} \int_0^x \frac{dx'}{u(x')} \right] 1\left(t - \int_0^x \frac{dx'}{u(x')} \right)$$




constant current
increased flow
decreased flow
Concentration
1
0.5
0
0
1
2
3
4
5
Distance
Flow
2
1
0
0
0.5
1
Distance

Unsteady Diffusion Process: No Flow

$$C(x,t) = C_0 \, erfc\left(\frac{x}{2\sqrt{Dt}} \right)$$



Mathematics:
∂C/∂x satisfies to fundamental solution