Runup of Destructive Waves on a Beach

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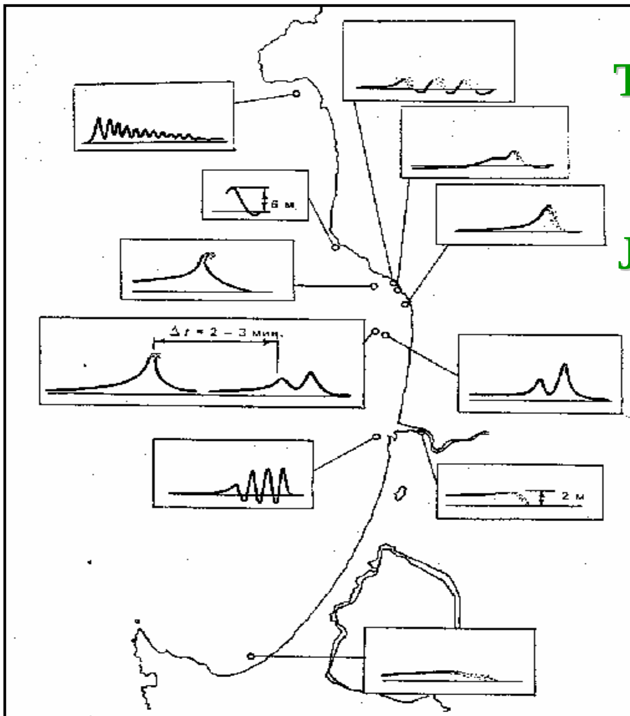
Long Wave Runup

Waves induced by high-speed ferries

Outline

- Runup of symmetric long waves on a beach: influence of the wave shape
- Runup of asymmetric long waves on a beach: influence of the wave steepness
- Runup on the beach of special profile (“nonreflecting” beach): traveling wave solutions



Tsunami Wave Shapes at Japanese Coast

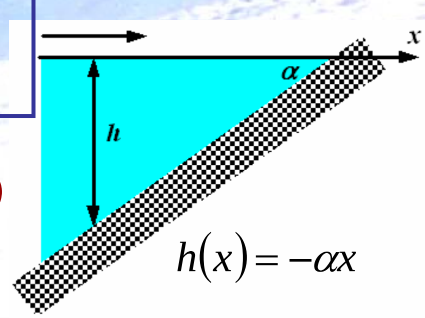
26 May 1983 Japan Sea (Shuto, 1985)

Nonlinear Shallow Water Theory

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left[(-\alpha x + \eta)u\right] = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0$$

(Carrier & Greenspan, 1958)



Hodograph Transformation

(Carrier & Greenspan, 1958)

$$\frac{\partial^2\Phi}{\partial\lambda^2}-\frac{\partial^2\Phi}{\partial\sigma^2}-\frac{1}{\sigma}\frac{\partial\Phi}{\partial\sigma}=0$$

$$\sigma = 2\sqrt{g(h+\eta)} \geq 0$$

Implicit form

$$\eta=\frac{1}{2g}\left(\frac{\partial\Phi}{\partial\lambda}-u^2\right)$$

$$u=\frac{1}{\sigma}\frac{\partial\Phi}{\partial\sigma}$$

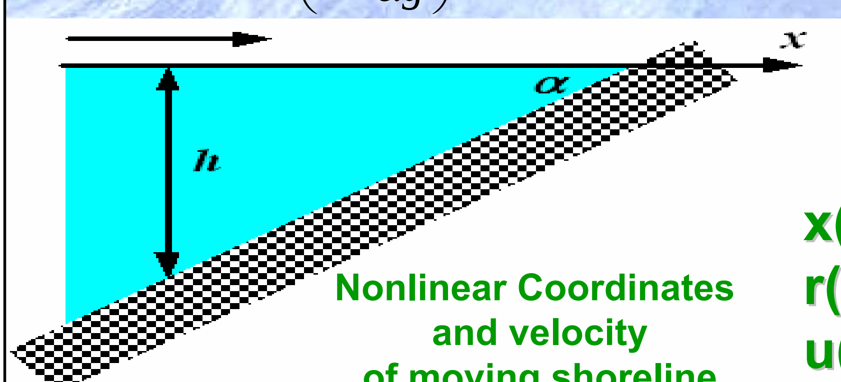
$$x=\frac{1}{2\alpha g}\left(\frac{\partial\Phi}{\partial\lambda}-u^2-\frac{\sigma^2}{2}\right)$$

$$t=\frac{1}{\alpha g}\left(\lambda-\frac{1}{\sigma}\frac{\partial\Phi}{\partial\sigma}\right)$$

Explicit Solution for Moving Shoreline if Incident Wave is given Far from Shoreline where it is Linear

Pelinovsky & Mazova, 1992

$$u(t)=U\left(t+\frac{u}{\alpha g}\right) \qquad r(t)=\alpha\int u(t)dt$$



Nonlinear Coordinates
and velocity
of moving shoreline

x(t)
r(t)
u(t)

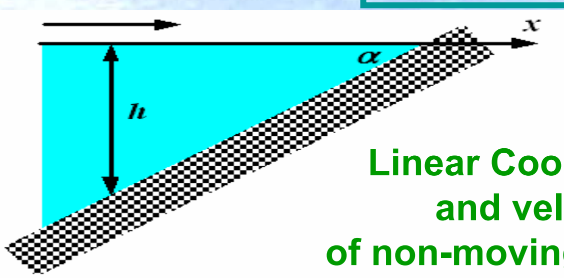
First Step – Solution of Linear Equations For Wave Transformation on a Beach

Incident Wave

$$\eta(t) = \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

$$R(t) = 2\pi\sqrt{\frac{2L}{\lambda}} \sum_{n=1}^{\infty} \sqrt{n} A_n \sin\left(n\omega t + \varphi_n + \frac{\pi}{4}\right)$$

$$U(t) = \frac{1}{\alpha}\frac{dR}{dt}$$

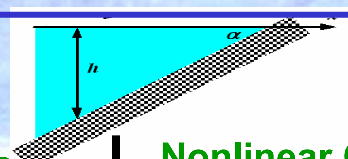


Linear Coordinates
and velocity
of non-moving shoreline

x=0
R(t)
U(t)

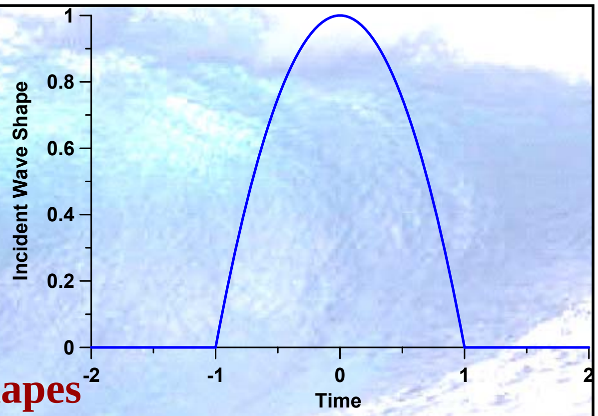
Second Step – "Nonlinear" Moving Shoreline

$$u(t) = U\left(t + \frac{u}{\alpha g}\right) \qquad r(t) = \alpha \int u(t)\,dt$$



Linear Coordinates and velocity of non-moving shoreline	Nonlinear Coordinates and velocity of moving shoreline
x=0, R(t), U(t)	**x(t), r(t), u(t)**

Motivation



One-scale wave shapes are analyzed in literature

- Sine wave and sine pulse
- Lorentz and Gauss pulses
- Soliton and N-wave

Does Runup Height depend on the Incident Wave Shape where the Incident Wave is symmetrical?

For periodic wave – yes!

The anomalous behavior of the runup of cnoidal waves

Costas Emmanuel Synolakis and Manas Kumar Deb
School of Engineering, University of Southern California, Los Angeles, California 90089

James Eric Skjelbreia
MARINETEK, Hakonsongst 34, 7002 Trondheim, Norway

Phys. Fluids, 1988, v. 31, No. 1

For single wave???

Incident wave shapes used in literature:

1. **Solitary Wave** $f(\mathrm{x}) = \mathrm{sech}^2(\mathrm{x})$
2. **Gaussian Pulse** $f(x) = \exp(-x^2)$
3. **Lorentz Pulse** $f(\mathrm{x}) = \dfrac{1}{1+x^2}$

and several others

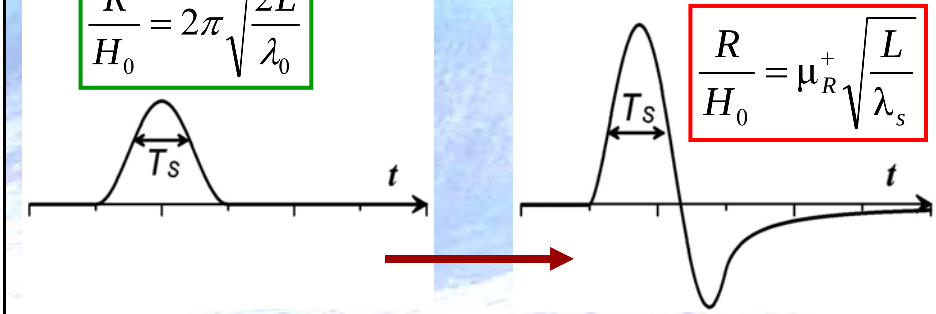
Wave Length (Duration) Definition for Pulse

Sine Wave

$$\frac{R}{H_0} = 2\pi\sqrt{\frac{2L}{\lambda_0}}$$

How this formula can be modified?

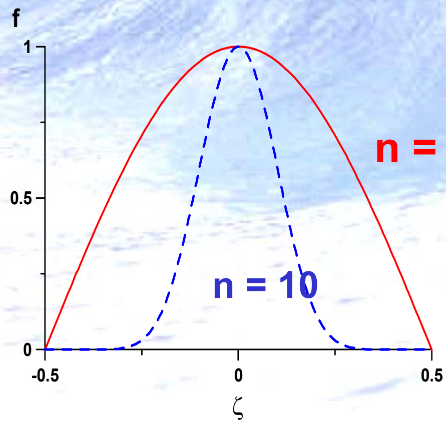
$$\frac{R}{H_0} = \mu_R^+\sqrt{\frac{L}{\lambda_s}}$$

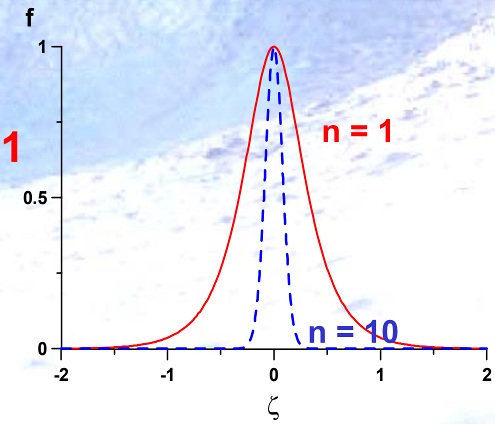


We suggest to use length of the wave on 2/3 level – philosophy used to define the significant wave properties

Incident Wave Shapes: examples

$$f(\mathrm{x}) = \cos^n(\pi \mathrm{x}) \qquad f(\mathrm{x}) = \mathrm{sech}^n(4\mathrm{x})$$

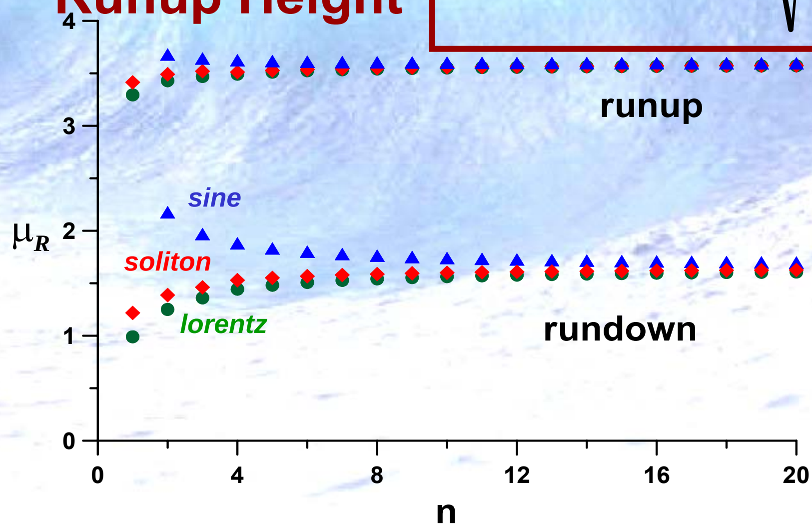


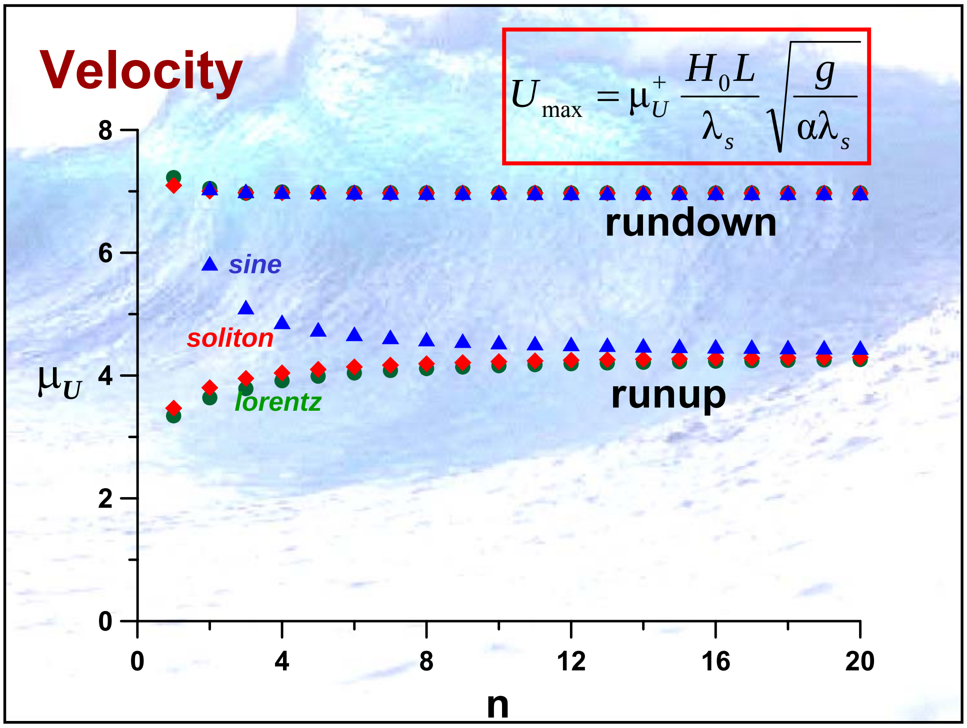


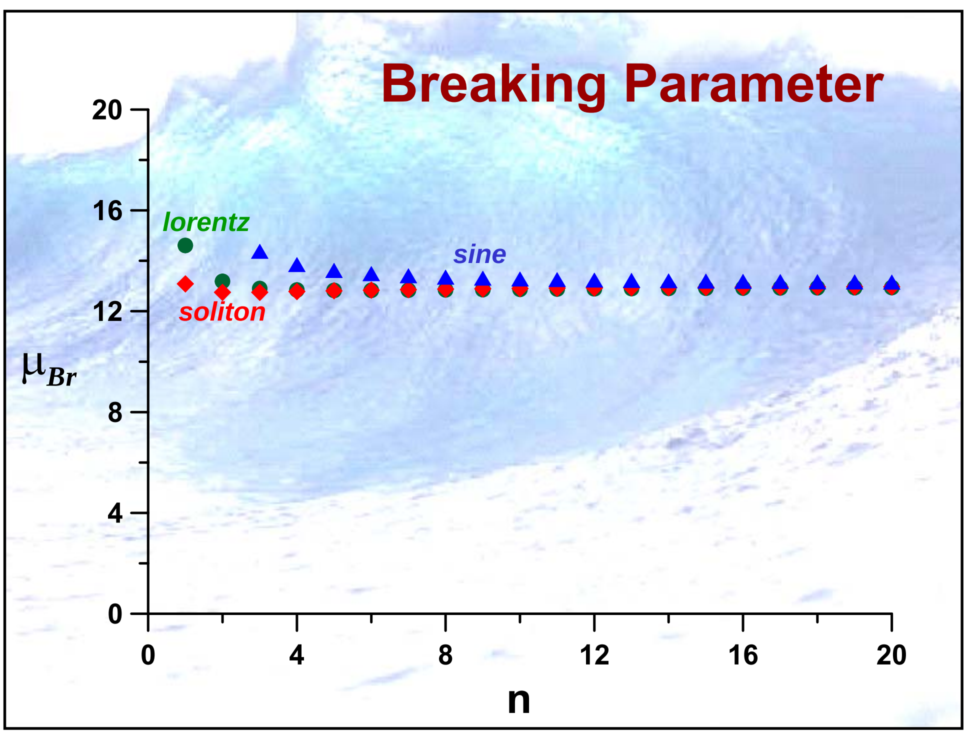
$$R = 2\pi H_0 \sqrt{\frac{2L}{\lambda_0}}$$

Runup Height

$$R_{\max} = \mu_R^{+} H_0 \sqrt{\frac{L}{\lambda_s}}$$




Velocity
$$U_{max} = \mu_U^+ \frac{H_0 L}{\lambda_s} \sqrt{\frac{g}{\alpha \lambda_s}}$$
rundown
sine
soliton
lorentz
runup
μU
n
0
2
4
6
8
0
4
8
12
16
20


Breaking Parameter
lorentz
sine
soliton
μBr
n
0
4
8
12
16
20
0
4
8
12
16
20

Parameterized Formulas

$$R_{runup} = 3.5H_0\sqrt{\frac{L}{\lambda_s}}$$

$$R_{rundown} = 1.5H_0\sqrt{\frac{L}{\lambda_s}}$$

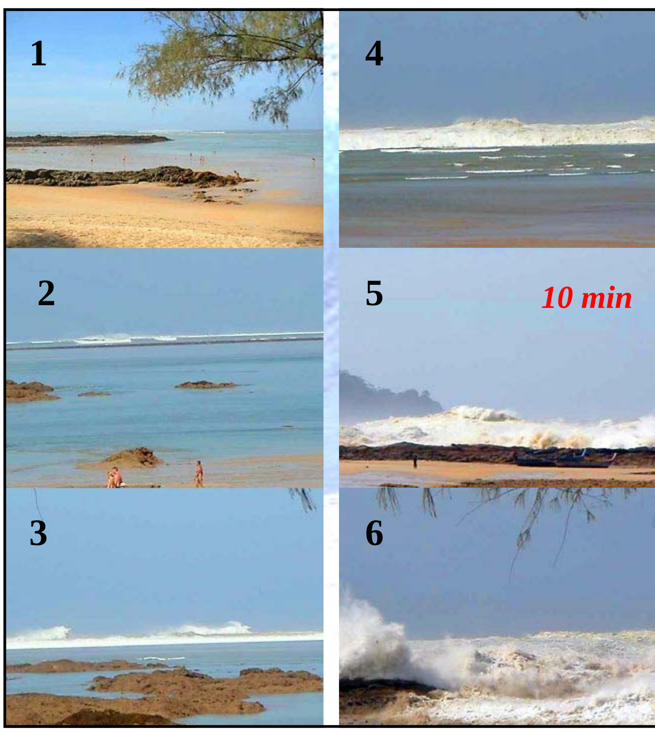
$$U_{runup} = 4.5\frac{H_0L}{\lambda_s}\sqrt{\frac{g}{\alpha\lambda_s}}$$

$$U_{rundown} = 7\frac{H_0L}{\lambda_s}\sqrt{\frac{g}{\alpha\lambda_s}}$$

$$Br = 13\frac{H_0L}{\alpha\lambda_s^{2}}\sqrt{\frac{L}{\lambda_s}}$$

Applications:

Fast Estimates of Runup Characteristics



Indonesian Tsunami 2004, December 26

Shock Wave

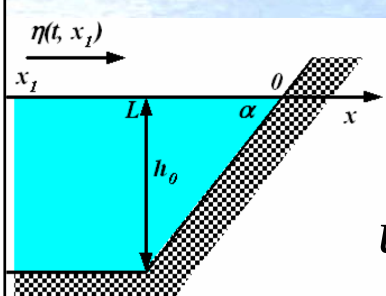
J&J Cnale
Canadian Couple

Incident wave can be asymmetric!
(two scales for face and back slopes)

Nonlinear Deformation

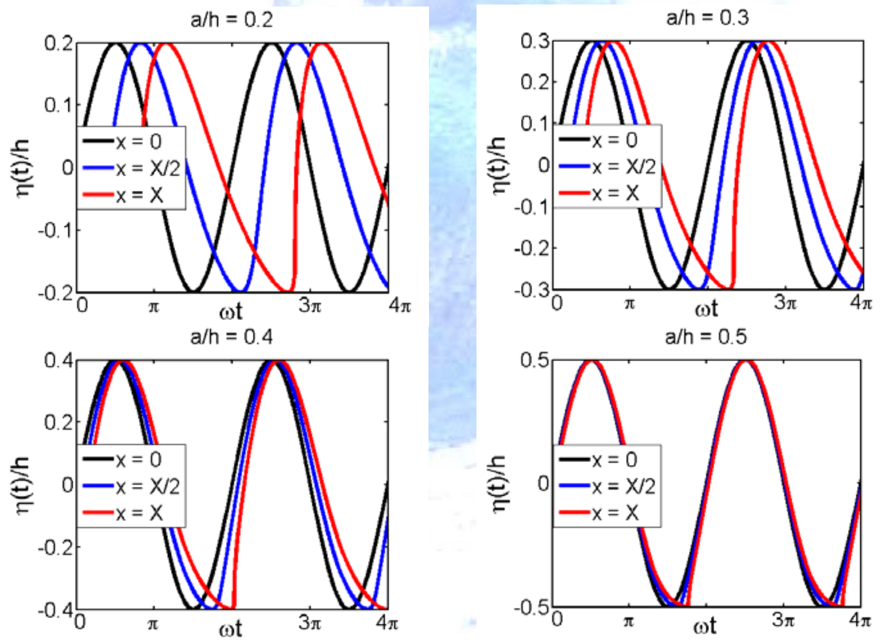
$$\frac{\partial \eta}{\partial t} + V\frac{\partial \eta}{\partial x} = 0$$

$$\eta(x,t) = \eta_0\left(t - \frac{x}{V(\eta)}\right)$$

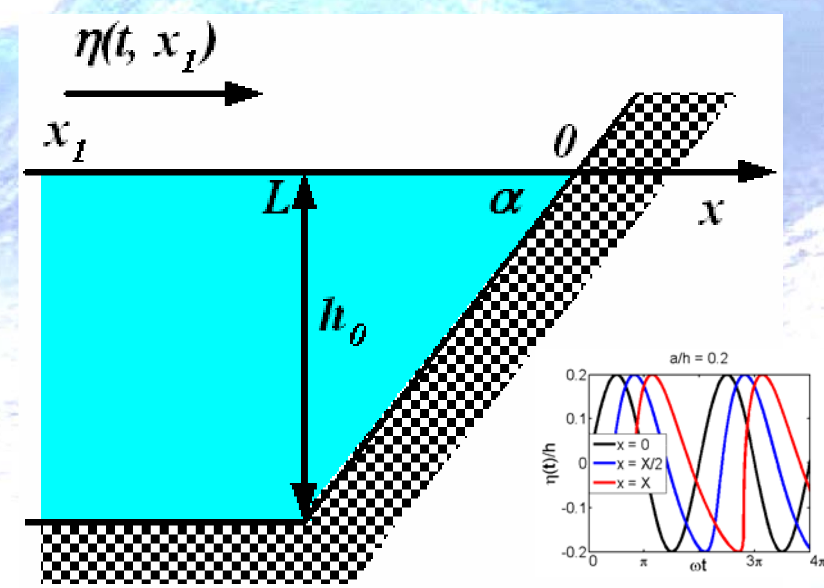


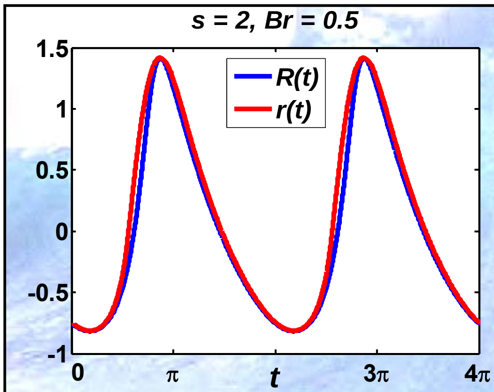
$$V = 3\sqrt{g(h+\eta)} - 2\sqrt{gh}$$

$$u = 2\left(\sqrt{g(h+\eta)} - \sqrt{gh}\right)$$



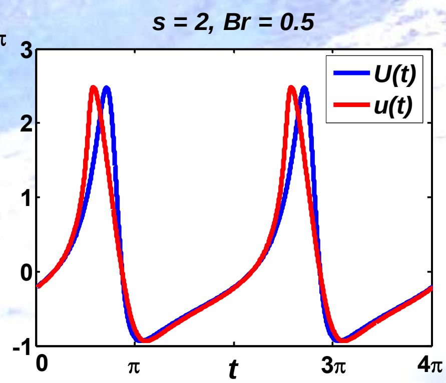
Nonlinear Deformed Wave as Input for Runup Formulas

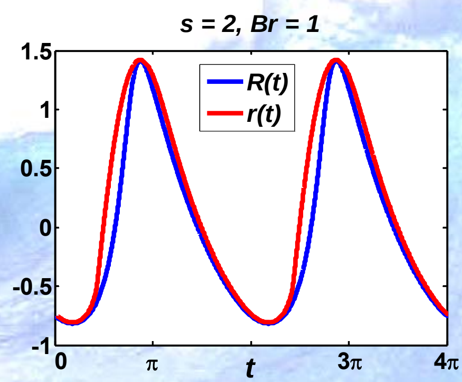




Runup of steep waves

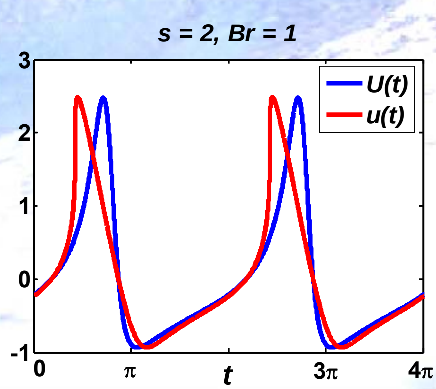
Moving shoreline

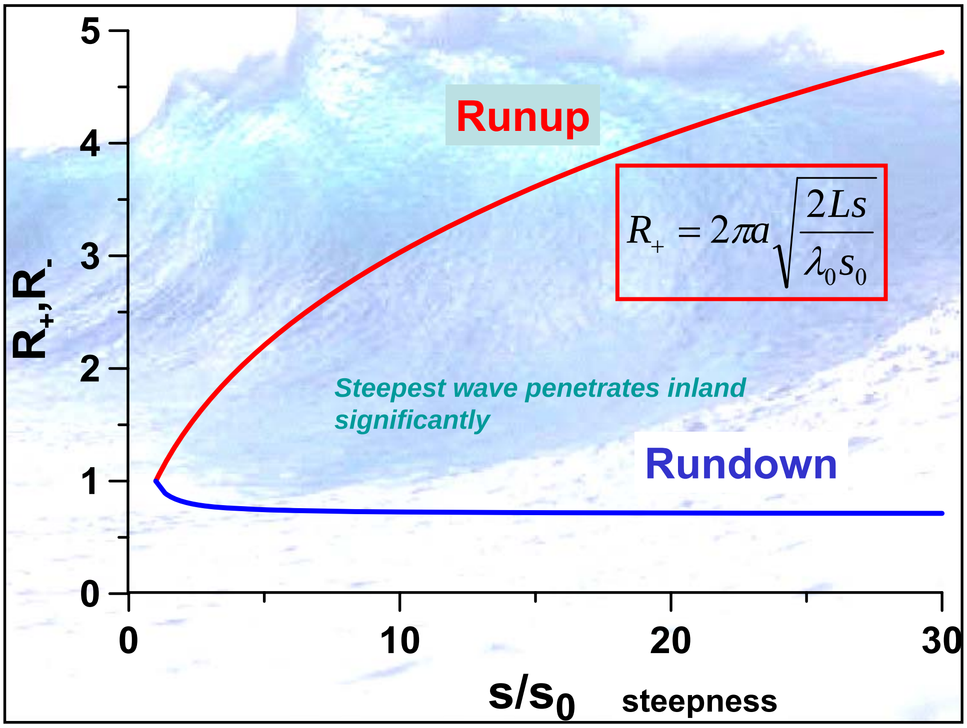




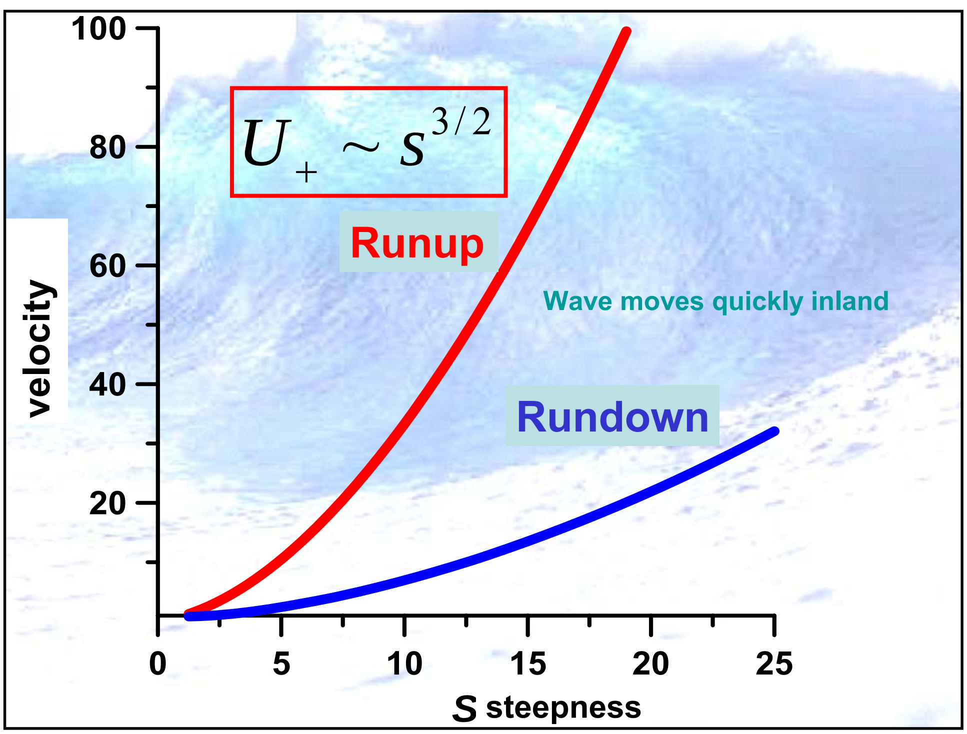
Runup of steep waves

Moving shoreline





Runup
$R_+ = 2\pi a\sqrt{\frac{2Ls}{\lambda_0 s_0}}$
Steepest wave penetrates inland significantly
Rundown
R_+, R_-
s/s_0 steepness



$U_+ \sim s^{3/2}$
Runup
Wave moves quickly inland
Rundown
velocity
S steepness

Conclusion:

Steep Wave Penetrates Inland
over Larger Distance and with Greater Velocity, than a symmetric one
-and Slowly into the Sea

Formulas for Solitary Wave Runup can be Parameterized

These results are important for engineering estimates

Long Wave Dynamics above Inclined Bottom of a Special Profile

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left[h(x)u\right] = 0$$

$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0$$

Linear shallow water theory

Wave Equation

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x}\left[c^2(x)\frac{\partial \eta}{\partial x}\right] = 0$$

$$c^2(x) = gh(x)$$

Traveling wave solutions for arbitrary bottom profile

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x}\left[gh(x)\frac{\partial \eta}{\partial x}\right] = 0$$

strongly inhomogeneous media

$\eta(x) = A(x)v(x,t)$

$x \to \tau(x)$

$A(x) = ? \quad \tau(x) = ? \quad h(x) = ?$

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial \tau^2} = 0$$

$A(x) \sim x^{-1/3}$

$\tau(x) \sim x^{1/3}$

$h(x) \sim x^{4/3}$

Boundary condition

First all, the wave equation should be solved on semi-axis ($0<\tau<\infty$)

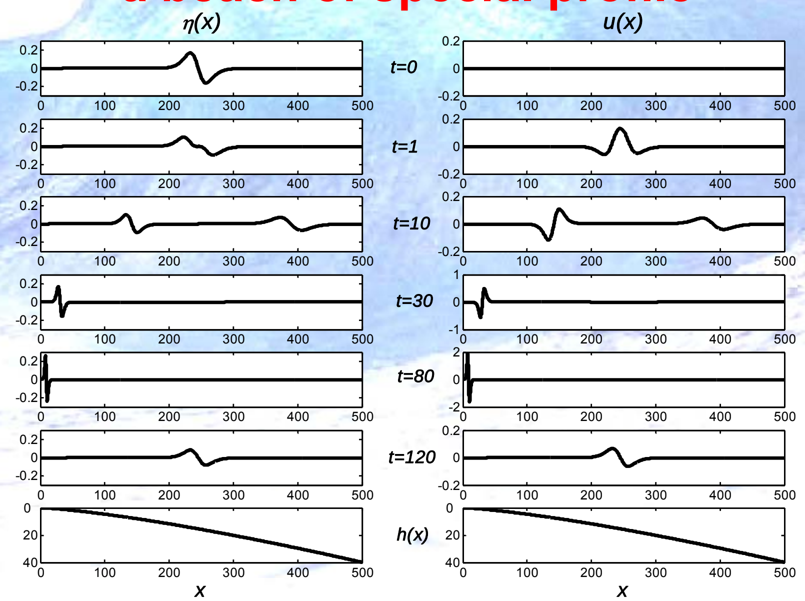
The natural boundary condition for the “reduced” wave equation in this point $\tau=0$ is

$$v(\tau = 0, t) = 0$$

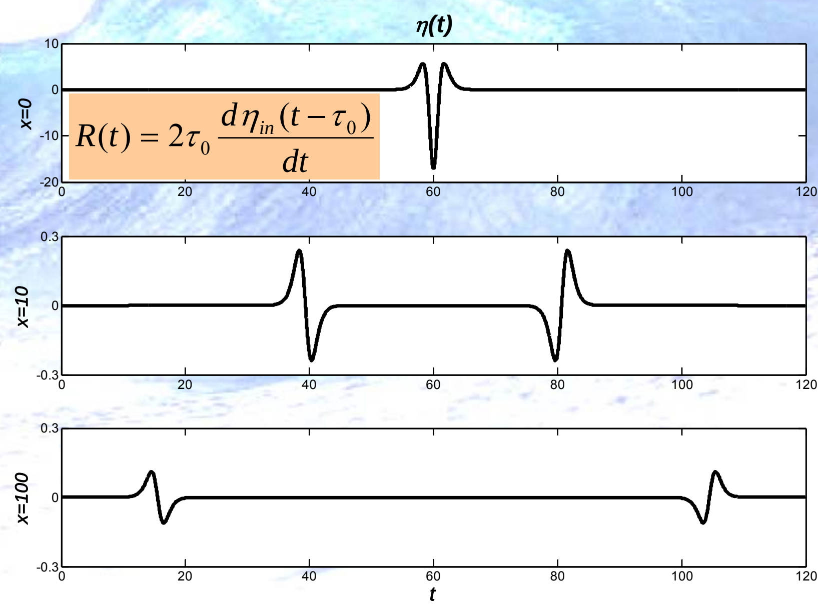
It provides the boundedness of the water displacement on the shore.

In this case the domain can be extended to the whole axis ($-\infty<\tau<+\infty$) and initial conditions should be continued for $\tau<0$ with sign inversion of the water displacement (“imaginary mirror” reflection condition).

Water displacement and velocity on a beach of special profile



The wave amplification when the wave approaches the shore and its differentiation on the shoreline



Maximal runup height

if an incident wave is the solution of the Korteweg – de Vries equation

$$\eta(t) = A\operatorname{sech}^2\left[\sqrt{\frac{3Ag}{4h^2}}\,t\right]$$

$$R_{\max} = 4L\left(\frac{A}{h}\right)^{3/2} = 4\frac{A}{\alpha}\sqrt{\frac{A}{h}} \sim A^{3/2}$$

for a beach of special profile

where α is the mean slope of a beach

$$R_{\max} = 2.8312\frac{A}{\sqrt{\alpha}}\left(\frac{A}{h}\right)^{1/4} \sim A^{5/4}$$

for a plane beach
Synolakis (1987)

Conclusions

- The runup of solitary waves of moderate amplitudes on a beach of special profile leads to more energetic amplification than for the beach of constant slope
- The shape of the water oscillations in the shoreline is determined by the first derivative of the incident wave shape